



Derivable geodesic weaving: enabling tension-compression anisotropic topology optimization of densified continuous fiber paths with controlled-spacing

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ABSTRACT

Designing continuous fiber-reinforced composites (CFRCs) with homogenized constitutive models is efficient at the macro-scale but yields abstract fields (e.g., density, orientation) rather than printable paths, thereby missing key manufacturing details. To bridge this gap, a dual-field co-optimization method following the DGTO (Derivable Geodesics-coupled Topology Optimization) framework is presented that preserves the efficiency by homogenization and enforces manufacturability by coupling spacing constraints and a dedicated post-processing pipeline for continuous path generation. Specifically, an orthogonal dual-field representation is introduced to produce a globally consistent vector field from geodesics, eliminating angle ambiguity and enabling continuous path layout along principal stress trajectories. The composite's tension-compression (bi-modulus) anisotropy is encoded via a weighted-compliance objective, promoting tensile load transfer while suppressing compressive response. A fiber density variable is also introduced that characterizes the fiber path spacing, enabling a dense fiber infill while preventing overlap. Benchmarks demonstrate improved path continuity, uniform and dense fiber trajectories, and tension-based stiffness enhancement compared to single-field approaches. Experimental results further confirm that explicitly accounting for tension-compression anisotropy in the design substantially improves the mechanical performance of the optimized structures.

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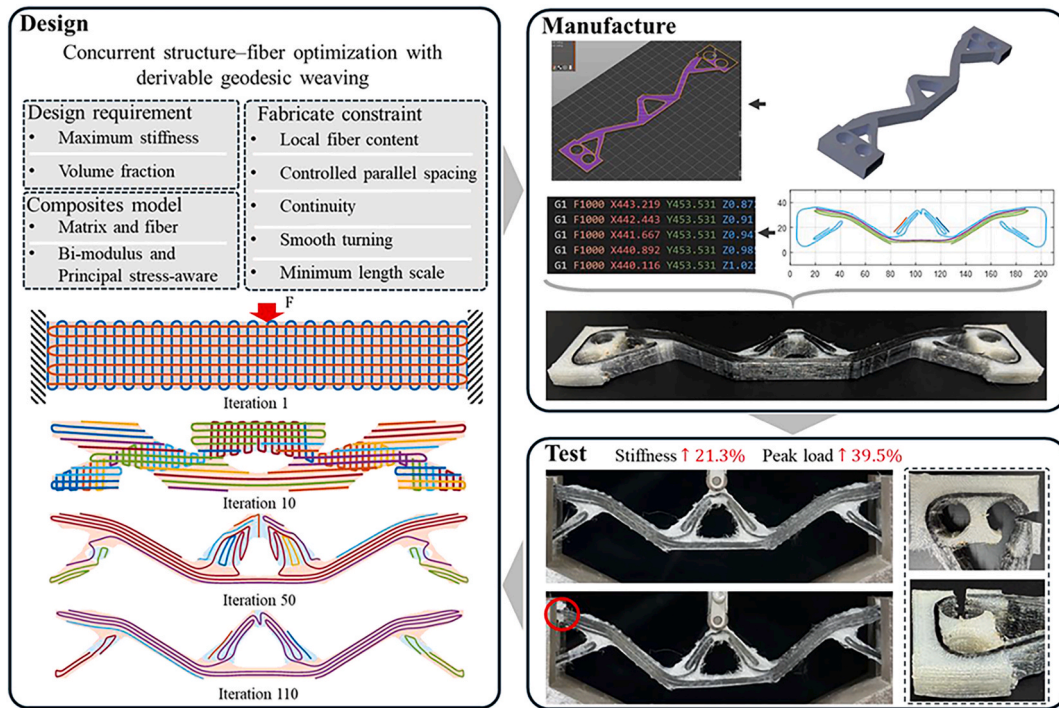
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fiber architecture to improve performance while preserving process feasibility.

1. Introduction

Continuous fiber-reinforced composites (CFRCs) achieve superior mechanical performance owing to the high strength and stiffness of the reinforcing fibers, while the matrix provides support, ensures effective load transfer, and offers protection against environmental degradation [1]. Crucially, these advantages are realized only when fiber directions and material placement are judiciously planned; with appropriate tailoring, CFRCs can lead to order-of-magnitude gains in specific stiffness/strength over conventional lightweight metals [2]. Traditionally, the variable space of fiber orientations in composite structures was limited due to the tape laying or weaving-type manufacturing method. Recent advances in fabrication technologies, especially additive manufacturing (shown in Fig. 1), enable the deposition of curved fiber paths and the adjustment of local fiber volume fractions. These developments allow mechanical behaviors such as strength, stiffness, bending, and torsional responses to be better aligned with structural design requirements, leading to more efficient exploitation of the composites' potential [3–5].

Against this backdrop, topology optimization (TO) has become a powerful tool for designing CFRCs. TO seeks the optimal material distribution within a prescribed domain under given boundary conditions and loads [6]. Over the past decades, several TO paradigms have matured, including density method [7], level-set method [8], the Bi-directional Evolutionary Structural Optimization (BESO) method [9] and moving morphable components (MMC) approaches [10]. With the advancement of additive manufacturing, TO has experienced renewed momentum by coupling design with process capabilities [11–13]. Regarding TO of CFRCs for additive manufacturing, the design space extends beyond structural layout to the co-optimization of fiber orientation fields and executable fiber paths [14–18]. By exploiting the intrinsic anisotropy of composites and embedding manufacturability constraints, these formulations jointly tailor structural topology and

2. Related works

Recent review articles [19–21] suggest that existing TO for CFRCs can be broadly classified into sequential and concurrent approaches. In sequential approaches, the structural topology and the fiber architecture are determined in separate stages. For example, Wang et al. proposed a load-dependent path-planning method in which the structural topology is first optimized, after which the fiber paths are generated according to the principal stress distribution using the Stress Vector Tracing algorithm [22]. While such approaches are intuitive and relatively straightforward to implement, they do not fully incorporate the

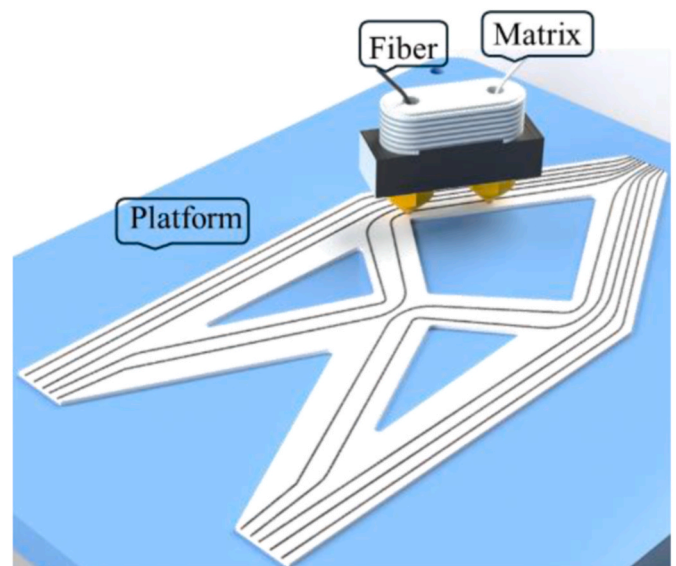


Fig. 1. Schematic of dual-nozzle CFRCs additive manufacturing.

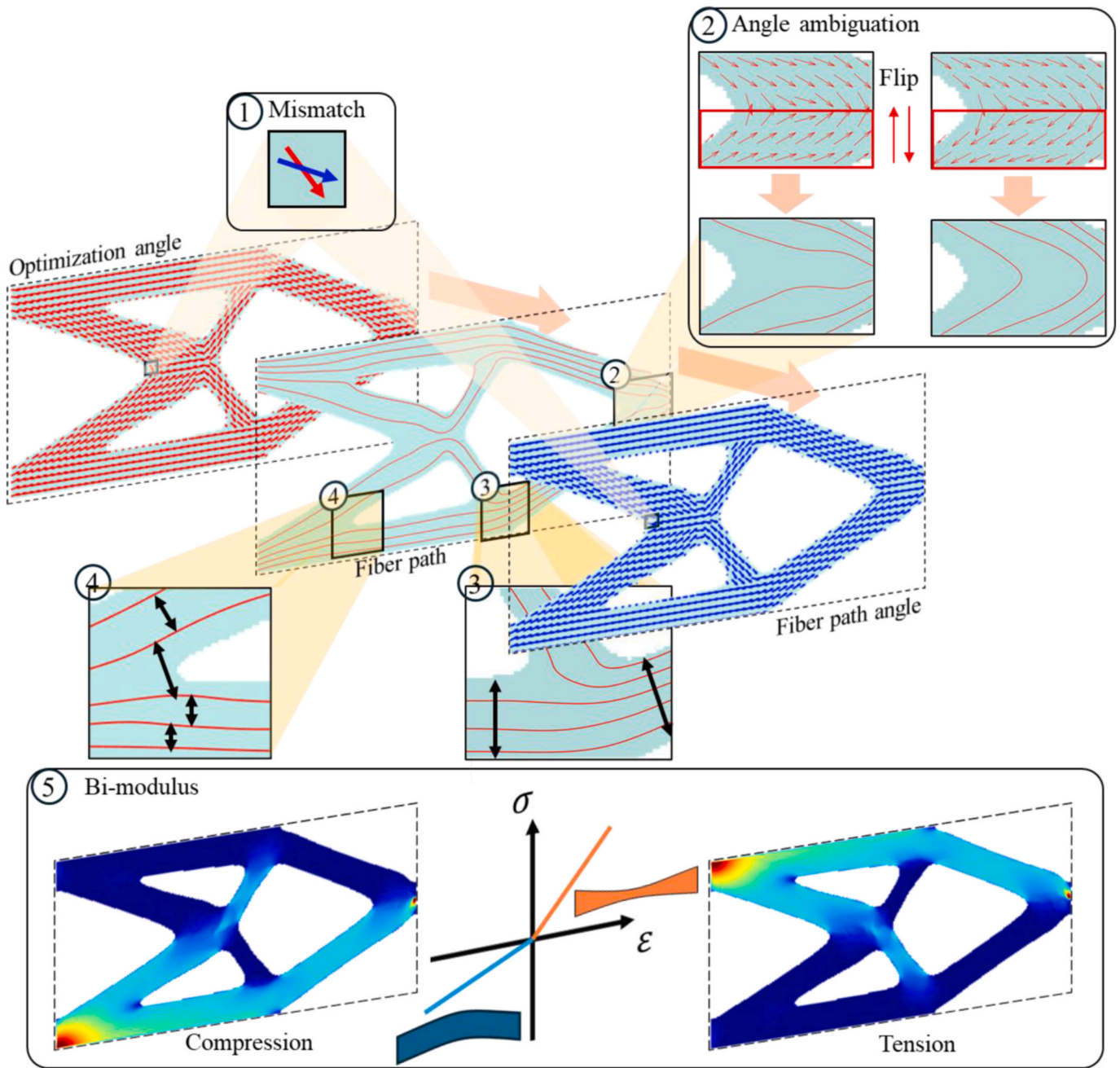


Fig. 2. Schematic of conventional discrete-orientation-based formulations, highlighting key pitfalls.

anisotropic effect of the fiber architecture during the TO process. Consequently, the coupling between structural layout and fiber reinforcement is only partially exploited, which may result in suboptimal structural performance.

By contrast, concurrent approaches optimize the structural topology and the fiber architecture simultaneously, thereby directly incorporating the anisotropic effect of fiber reinforcement into the optimization process and overcoming the principal limitation of sequential methods. Within this category, the representation of fiber paths plays a central role in determining the consistency between the optimized design and the finally manufactured structure. Existing studies can be further grouped into two main classes: discrete-orientation-based formulations and geometry-driven formulations. In the former, local fiber orientations are treated as independent design variables. This representation is concise, physically intuitive, and widely adopted in literature. For example, within the density-based TO framework, the Solid Orthotropic

Material with Penalization (SOMP) extends the Solid Isotropic Material with Penalization (SIMP) to orthotropic elasticity and has been widely used to optimize fiber distributions for high stiffness-to-weight applications [23–25]. In parallel, discrete material optimization (DMO) treats a set of candidate fiber orientations as distinct “materials” in a multi-material formulation, thereby reducing fiber-orientation optimization to a material-selection problem [26–28]. Similar discrete-orientation representations have also been adopted in the BESO framework, where they involved membrane-embedded models to account for geometrical nonlinearity [29,30], and in the geometry projection framework, where discrete orientations are assigned to different structural components [31–33]. In addition, recent machine-learning-assisted TO methods for CFRCs design also commonly predict discrete orientation fields [34,35].

Despite their practicality and widespread use, discrete-orientation-based formulations still exhibit several limitations: (i) Orientation con-

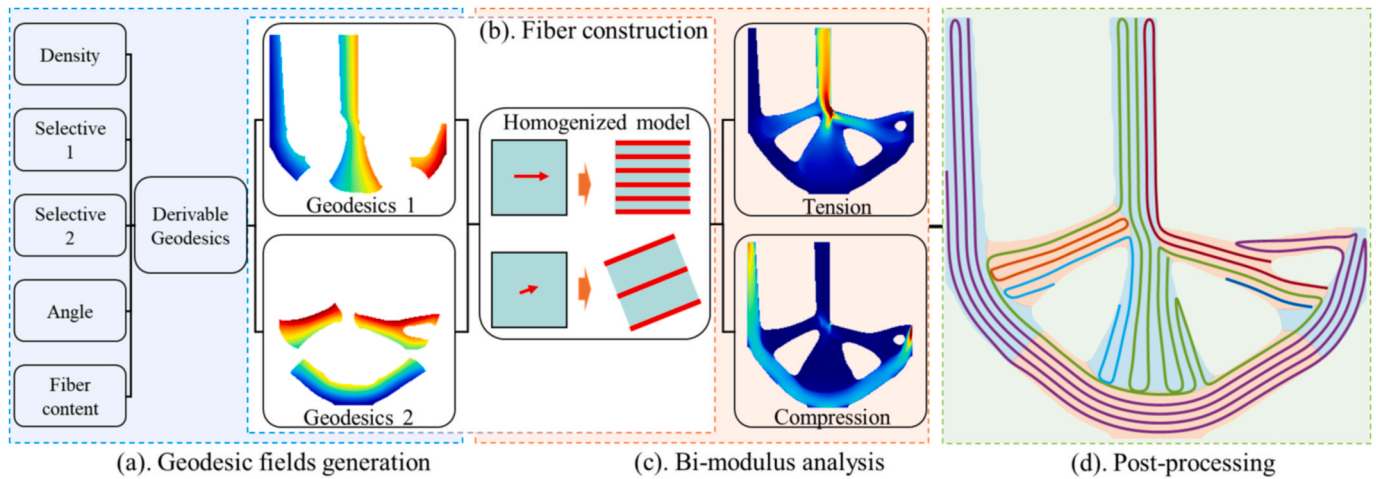


Fig. 3. Overview of the proposed dual-field framework. (a) Geodesic-field generation. (b) Fiber construction via a homogenized model. (c) Bi-modulus analysis (tension vs. compression). (d) Post-processed continuous fiber paths ready for fabrication.

tinuity. Because orientations are chosen from a discrete set, neighboring elements can exhibit abrupt orientation jumps. Various smoothing strategies (e.g., hybrid strain–stress formulations [36], filtering [37,38], or constraints [39,40]) have been introduced to promote spatially coherent orientation fields. (ii) Orientation ambiguity (π -periodicity). For orthotropic plies, θ and $\theta + \pi$ represent the same physical state, which can lead to direct conflicts in adjacent orientations that can trap the solution in a local minimum. To mitigate this issue, some studies constrain the admissible orientation range or project orientations into a reduced fundamental domain (e.g., Generalized Shape Function-based Parameterization (GSFP) [41], Bi-value Coding Parameterization (BCP) [42], Discrete-Continuous Parameterization (DCP) [43,44]). (iii) Inconsistency with manufacturing fiber paths. These discrete-orientation frameworks still exhibit a substantive gap from the final fiber paths for manufacturing. In practice, they rely on post-processing to convert elementwise orientations into continuous printable toolpaths, for example through the equally spaced (EQS) method [45], streamline tracing [46], geometric offsetting [47], or scalar-field projection methods [48]. During this post-processing stage, mismatches would arise between the optimized local fiber orientations and the resulting continuous fiber paths. Consequently, the TO process utilized orthotropic material properties that are not consistent with the fabricated result, indicating compromised optimality. Moreover, because path generation is performed in a separate stage, manufacturing capabilities and constraints are difficult to incorporate directly into the TO process.

To avoid such mismatches, the most direct strategy is to generate fiber paths during the optimization process rather than in a separate post-processing stage. This naturally leads to geometry-driven formulations, in which fiber paths are represented more directly as continuous geometric entities. For example, Li et al. proposed a method based on a local volume constraint to achieve a full-scale representation of composite structures, in which both the fiber paths and the matrix material are explicitly modeled during optimization [49]. Subsequently, Ren et al. further advanced this full-scale framework by incorporating scalar-field projection directly into the optimization procedure to generate fiber paths in a more integrated manner [50,51]. Although such full-scale TO frameworks can largely avoid the mismatch associated with post-processing-based path generation, their computational cost becomes prohibitive when detailed fiber-matrix interactions must be explicitly resolved.

Another important branch of geometry-driven formulations is the level-set-based approach, in which continuous fiber paths are represented mathematically by the iso-contours of a level-set function, rather than by the element-wise orientation field. In such formulations, the

local fiber orientation is typically defined by the tangent direction of the level-set iso-contour, which establishes a more direct link between the optimization model and the resulting continuous fiber paths [52–54]. Nevertheless, existing level-set-based methods still have several limitations. In many such formulations, the resulting fiber paths are essentially generated by inward offsetting of the structural boundary, so that the path distribution is governed primarily by geometric boundary features rather than by the intrinsic mechanical demand inside the structure. Such fiber paths are also problematic at the peak areas of the level set function, where voids or overlaps can be encountered that degrade the fiber infill quality.

In addition, the tensile and compressive responses of 3D-printed CFRCs along the fiber direction may also differ significantly, indicating a pronounced bi-modulus behavior. This tension-compression asymmetry has been reported in the literature. For example, Hou et al. systematically characterized the tensile and compressive properties of 3D-printed CFRCs with different fiber volume fractions and established a corresponding constitutive model [55]. Their experimental results clearly demonstrated a pronounced tensile-compressive asymmetry. However, only a limited number of TO studies have explicitly taken this effect into account [56]. Under such circumstances, if fiber paths are generated primarily through geometric boundary offsetting, they may deviate from the principal tensile load paths and may even be distributed transversely across the main load-bearing skeleton. Therefore, for material systems that are both strongly anisotropic and tension-compression asymmetric, a path-generation strategy based solely on geometric boundary offsetting is unlikely to adequately reflect the true mechanical demand of the structure.

To address the above limitations, this work proposes a dual-field co-optimization method for structural topology and continuous fiber paths, following the DGTO (Derivable Geodesics-coupled Topology Optimization) framework [57–59]. The proposed method aims to simultaneously maintain geometric continuity and manufacturability while improving the mechanical consistency between the generated fiber paths and the principal stress directions of the structure. In addition, the tension-compression asymmetry of the material is incorporated directly into the optimization process, thereby establishing a unified coupling between geometric representation and physical response. The main contributions are summarized as follows:

- **Geodesics-based path-generation pipeline.** A geodesics-based processing pipeline is developed to generate fabrication-ready continuous fiber toolpaths during optimization. Here, *geodesics* denote a scalar field constructed through the Poisson framework [60], whose iso-contours represent the fiber paths while the geodesic distance

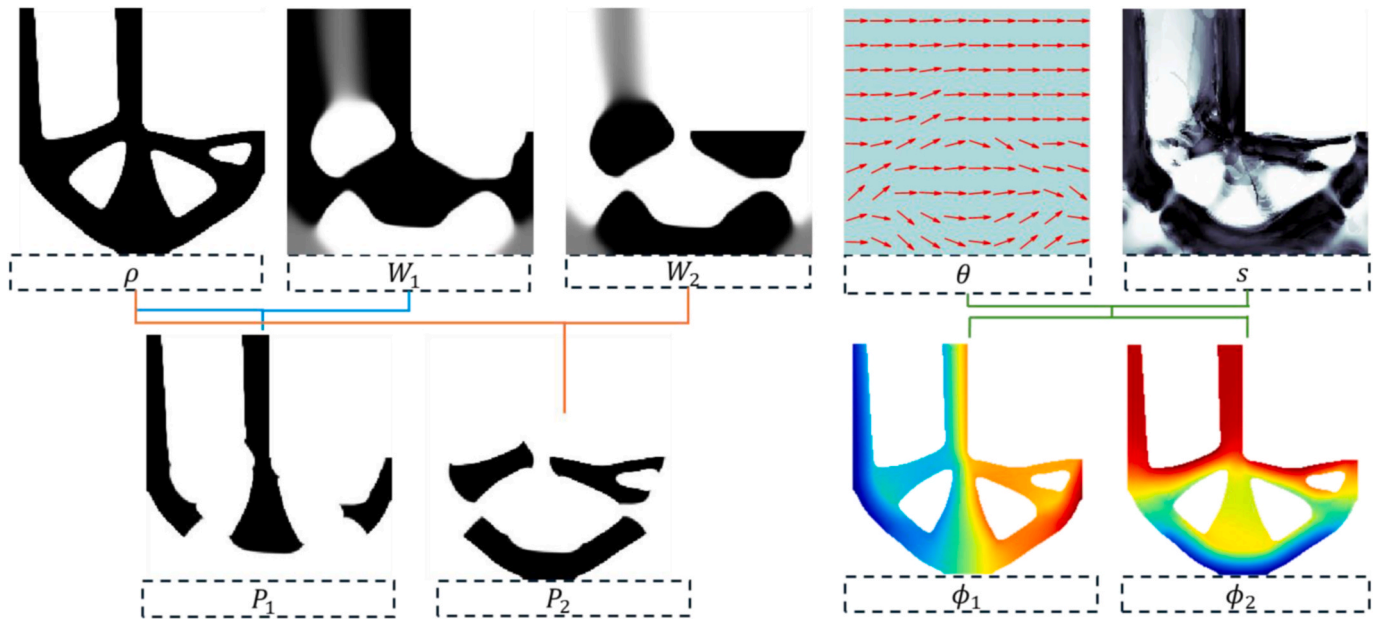


Fig. 4. Pipeline for constructing the two geodesic fields. From left to right: the density field ρ , in combination with the phase variables W_1, W_2 (derived from selective variables x_1, x_2), produces two phase masks; the angle variable θ defines the vector field (red arrows), and together with the spacing variable s , parameterizes the Poisson equation that yields the two geodesic potentials ϕ_1, ϕ_2 . (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

indicates the spacing between paths. The geodesic field is adopted, instead of the Euclidean distance field, for two reasons: the distance field generation is performed coupled with a complex topological structure that the straight line distance is not always accessible, and the current method is readily to be extended to optimizing continuous fiber paths on curved layers for five-axis 3D printing.

- Ambiguity-robust fiber paths are modeled via dual-fields. Two differentiable geodesic fields with constrained gradients are introduced, by which a vector field is constructed whose induced trajectories are globally consistent. Through this strategy, the π -periodic orientation ambiguity is resolved, and the orientation continuity is improved across the domain, facilitating manufacturable path generation.
- Bi-modulus constitutive modeling. A weighted-compliance-matrix (WCM)-based anisotropic bi-modulus constitutive model is incorporated into the optimization process, enabling the finite element analysis to distinguish tensile and compressive responses and providing a physically consistent basis for stress-guided fiber-path generation.
- Content-aware spacing control. A fiber-density variable is coupled with the geodesic field gradients so that printable fiber spacing is explicitly encoded during optimization. This establishes a direct coupling between the dual-scalar field and the material interpolation, thereby improving the consistency between path representation and mechanical response.

The remainder of this paper is organized as follows: Section 3 outlines the problem settings. Section 4 presents the methodology. Section 5 summarizes the optimization model. Section 6 conducts a sensitivity analysis. Section 7 presents numerical examples. Section 8 reports experimental validation and comparative studies. Section 9 concludes the paper and discusses future research directions.

3. Problem definition

This work aims to establish a framework for the concurrent optimization of structural topology and continuous fiber paths. Fig. 2 provides an intuitive illustration of the issues addressed in the present work:

- 1 During post-processing, mismatches may arise between the optimized local fiber orientations and the resulting continuous fiber path directions.
- 2 Because orthotropic plies are π periodic. A 180° flip within a neighborhood therefore leaves the constitutive response unchanged but can yield topologically different paths. As highlighted by the red boxes, two fields that are materially equivalent generate distinct outcomes: one side produces separated paths, while the other preserves continuity.
- 3 Imbalanced filling density in converging/diverging branches. When continuous paths are traced, converging branches of equal thickness inevitably collect more streamlines than diverging branches, leading to insufficient fiber content of the branching members.
- 4 Missing control of path spacing. Standard procedures of streamline generation do not parameterize spacing between paths or local percentage of fiber content. As a result, the spacing between neighboring paths varies that plays against the uniformity assumption of the element-level fiber definition.
- 5 Regions dominated by tensile versus compressive stresses exhibit bi-modulus behavior in 3D-printed CFRCs. Current continuous path-optimization schemes typically ignore this anisotropy.

To overcome these limitations, a dual-field co-optimization scheme that embeds spacing control and bi-modulus constitutive modeling, yielding ambiguity-free and manufacturable fiber paths with improved structural performance is developed.

4. Methodology

The overall framework is summarized in Fig. 3, which comprises four modules: (1) Geodesic-field generation, (2) Fiber-model construction, and (3) Tension-compression (bi-modulus) analysis, followed by (4) post-processing that converts the optimized fields into printable continuous fiber paths. The following subsections detail each module.

4.1. Generation of geodesic fields

The first module concerns the generation of geodesic fields and in-

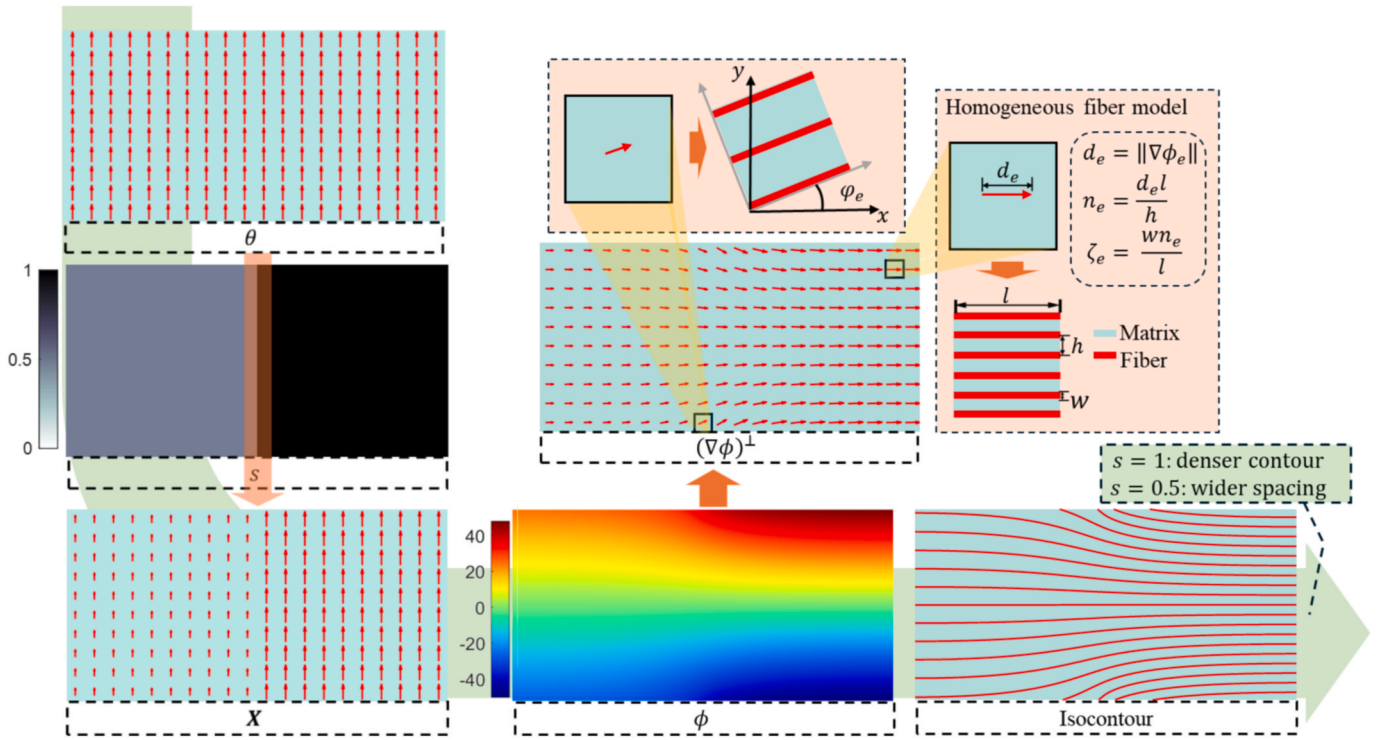


Fig. 5. Continuous fiber path generation and homogenized fiber model construction. (Note: $(\nabla\phi)^\perp$ denotes the vector orthogonal to $\nabla\phi$, i.e., a 90° rotation in 2D.)

volves five design variables: density (ρ), selective variables (x_1, x_2), an angle variable (θ), and a spacing variable (s). The density variable governs the presence or absence of material. The selective variables determine which geodesic field a given element belongs to. The angle variable defines the gradient direction of the geodesic field, while the spacing variable controls the interval of the geodesic field. The detailed procedure of this first module is illustrated in Fig. 4.

Here, W_1 and W_2 are obtained from the selective variables by the following expression:

$$W_i = \frac{\omega_i}{\omega_1 + \omega_2}, i = 1, 2; \omega_1 = x_1^q (1 - x_2^q); \omega_2 = x_2^q (1 - x_1^q) \quad (1)$$

where q is the penalization parameter, following the DMO scheme [26]. After partitioning the domain, coupling with the density variable yields the two phases P_1 and P_2 :

$$P_i = \rho W_i \quad (2)$$

On the right side of Fig. 4, the two geodesic fields ϕ_i are constructed from the angle and spacing design variables by solving a Poisson equation:

$$\Delta\phi_i = \nabla \cdot \mathbf{X}_i \quad (3)$$

where $\mathbf{X}_i$ is the vector:

$$\mathbf{X}_i = s \begin{bmatrix} \cos(\theta_i) \\ \sin(\theta_i) \end{bmatrix}, \theta_1 = \theta; \theta_2 = \theta + \frac{\pi}{2}; \theta \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right] \quad (4)$$

The formulation of generating two orthogonal geodesic fields is adopted to avoid orientation ambiguity. During optimization, the angle variable θ is restricted to $\left[-\frac{\pi}{4}, \frac{\pi}{4}\right]$, thereby preventing the π periodic ambiguity, and this angle is mapped to two orthogonal fields θ_1 and θ_2 . As a result, the effective admissible range of fiber orientations becomes $\left[-\frac{\pi}{4}, \frac{3\pi}{4}\right]$, still covering the full feasibility of orientations. The above formulation is similar to the two-subinterval strategy used in DCP [43]. The formulation of using dual fields also mitigates Issue ③ as depicted in

Fig. 2. The converging/diverging regions are split into two phases, while the amount of fiber trajectories are independently defined for each phase, thereby reducing the imbalance in fiber density between branching components.

4.2. Fiber model construction

This subsection introduces the fiber model built via homogenization. At each point, the fiber orientation is taken orthogonal to the gradient of the corresponding geodesic field, and the fiber density is quantified by the gradient magnitude: larger magnitudes imply faster field variation, denser isocontours, and therefore, higher local fiber content. With orientation and density specified, the composite is homogenized to obtain effective properties. The procedure is summarized in Fig. 5.

In Fig. 5, the green workflow arrow (background) highlights how the spacing variable s scales the vector-field magnitude, thereby controlling the isocontour spacing (i.e., the fiber interval). When $s = 0.5$ (vs. $s = 1$), the isocontour field is scaled down, the resulting geodesic potential varies slowly, and its isocontours (i.e., continuous fiber paths) are spaced about twice as far apart. Thus, decreasing s increases the path interval in reverse proportion.

The top right of Fig. 5 gives the details of the homogenized fiber model. Let d_e denote the magnitude of the gradient of the geodesics in element e , and let φ_e be the direction of the vector orthogonal to this gradient:

$$d_e = \|\nabla\phi_e\|, \varphi_e = \text{atan2}(-\phi_{x,e}, \phi_{y,e}) \quad (5)$$

Regarding details, l is the element length (square element), and h represents the default spacing between neighboring isocontours. Thereby, the number of fibers passing an element can be approximated by:

$$n_e = \frac{d_e l}{h} \quad (6)$$

Given a fiber width w , the fiber fraction ζ_e in element e is approximated by:

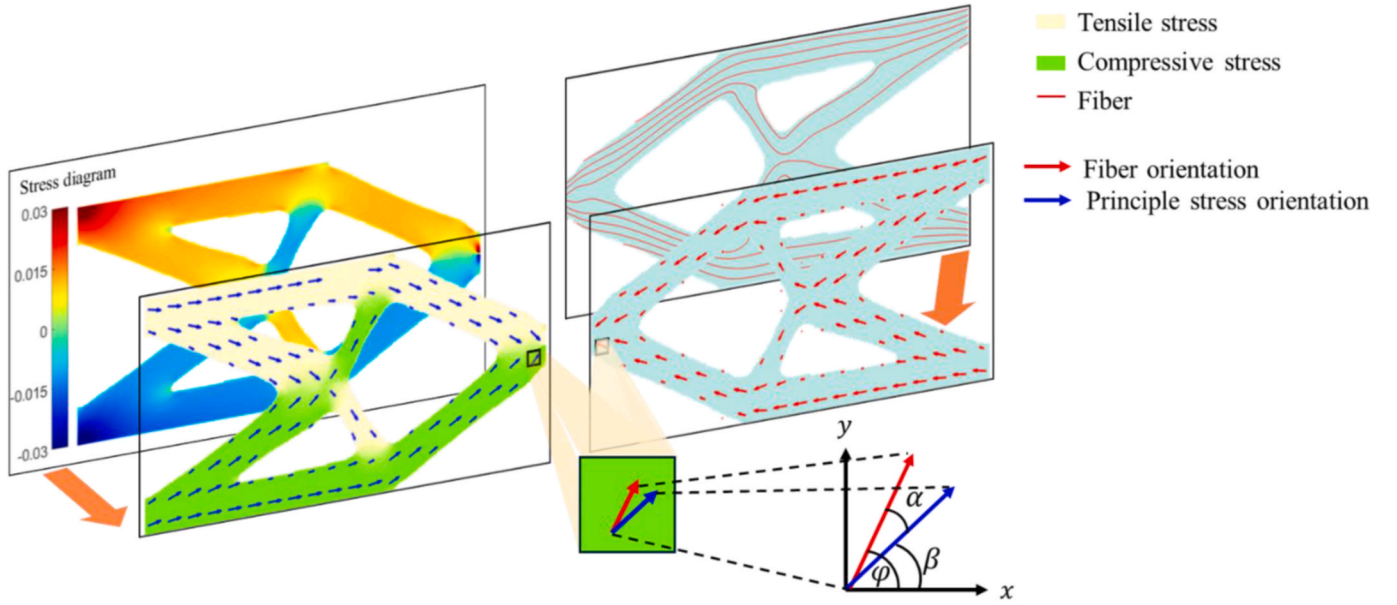


Fig. 6. Bi-modulus analysis coupled with fiber path.

$$\zeta_e = \frac{wn_e}{l} \quad (7)$$

If the fiber width equals the spacing between isocontours ($w = h$), one obtains the simple relation

$$\zeta_e = d_e \quad (8)$$

Clearly, the fiber fraction ζ_e should not exceed 1. Moreover, due to the current limitations of continuous-fiber 3D printing processes, the practically achievable fiber content is typically no more than 50% [61]. To account for this process constraint, an upper bound on the maximum value of d_e denoted $d_{max}(\theta, s)$ is introduced.

$$d_{max}(\theta, s) \leq d_0 \quad (9)$$

where d_0 is a process-driven limit on the maximum admissible value of d_e . To reduce computational cost, d_{max} is approximated by a p -norm aggregation:

$$d_{max} = \left(\sum_{i=1}^2 \sum_{e=1}^N (d_{i,e})^\mu \right)^{\frac{1}{\mu}} \quad (10)$$

where $d_{i,e}$ is the fiber fraction measure of phase i in element e , N is the number of elements, and μ controls the tightness of the max-approximation (set to 40 in this work). With fiber fraction determined, an interpolated constitutive model is used [62,63], through which the unrotated effective elastic constants are:

$$\begin{cases} E_{1,e} = E_1^f d_e + E_1^m (1 - d_e) \\ E_{2,e} = E_2^f E_2^m / (E_2^m d_e + E_2^f (1 - d_e)) \\ G_{12,e} = G_{12}^f G_{12}^m / (G_{12}^m d_e + G_{12}^f (1 - d_e)) \\ \nu_{12,e} = \nu_{12}^f d_e + \nu_{12}^m (1 - d_e) \\ \nu_{21,e} = (E_{2,e} / E_{1,e}) \nu_{12,e} \end{cases} \quad (11)$$

where the superscripts f and m denote the constituent parameter sets related to fiber and matrix of the interpolation model, respectively. It should be emphasized that these parameters should not be interpreted as the intrinsic elastic constants of the pure fiber and matrix constituents. Instead, they are effective parameters introduced in the interpolated constitutive model to reflect the overall mechanical response of the 3D-

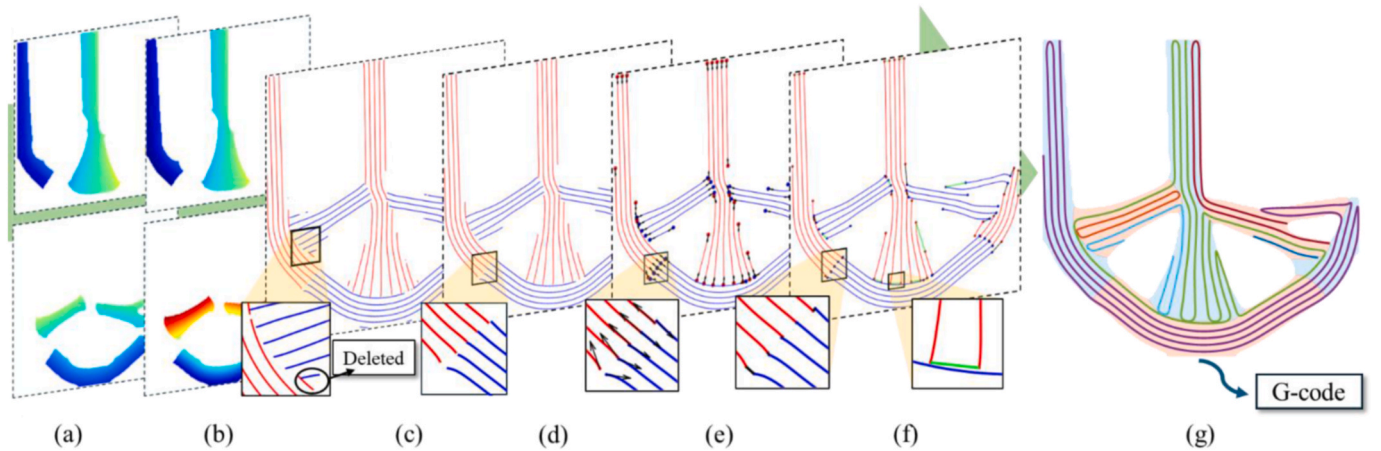


Fig. 7. Post-processing pipeline for the L-bracket. (a) Optimized geodesic dual fields; (b) geodesic fields after field alignment; (c) raw fiber paths extracted as isocontours; (d) pruning of short paths; (e) labeling of endpoints and directions; (f) connection of fibers, inter-phase (black) and inner-phase (green); (g) B-spline smoothing to obtain the final toolpaths. (For interpretation of the references to colour in this figure legend, the reader is referred to the Web version of this article.)

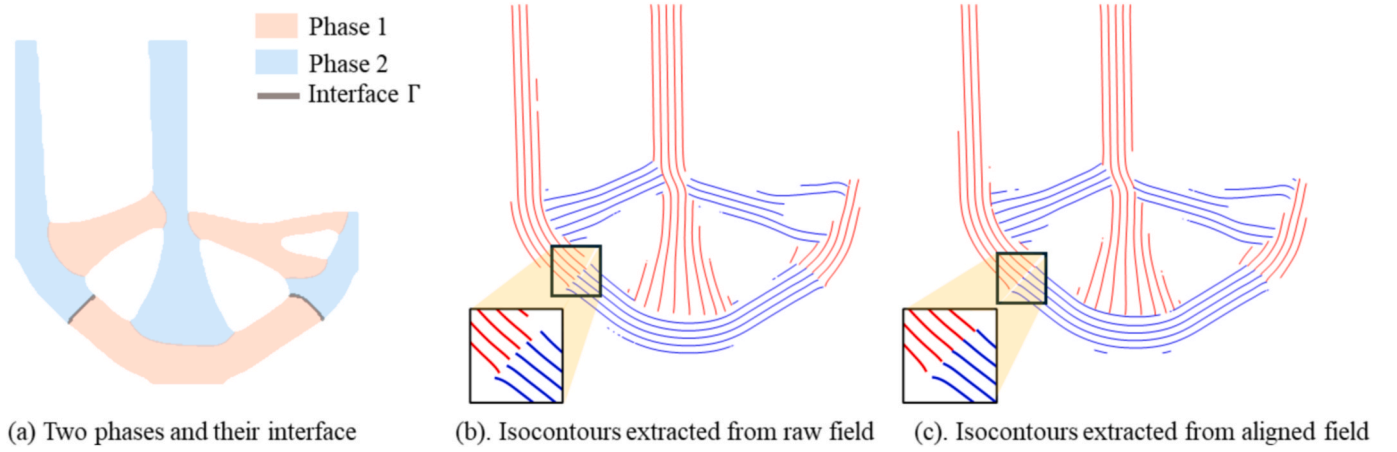


Fig. 8. Field alignment. (a) Two phases and their interface; (b) Isocontours extracted from raw field; (c) Isocontours extracted from aligned field.

printed CFRC, which is also influenced by printing-induced porosity, interfacial imperfections, and other manufacturing defects [55].

4.3. Bi-modulus analysis

After the effective elastic constants are assembled by Eq. (11), the effective moduli should be updated during loading according to the local stress state (tension vs. compression). The bi-modulus analysis adopted here follows the weighted WCM model [64]. According to the WCM model, the effective compliance coefficients in $\mathbf{S}^{pq}$ (inverse of the elastic matrix) are determined by the principal stress state as follows:

$$\begin{aligned}
 &\text{If } \sigma^p \geq 0, \sigma^q \geq 0 : \mathbf{S}^{pq(ij)} = \mathbf{S}_t^{pq(ij)} \\
 &\text{If } \sigma^p \leq 0, \sigma^q \leq 0 : \mathbf{S}^{pq(ij)} = \mathbf{S}_c^{pq(ij)} \\
 &\text{If } \sigma^p > 0, \sigma^q < 0 : \mathbf{S}^{pq(1,1)} = \mathbf{S}_t^{pq(1,1)}, \mathbf{S}^{pq(1,2)} = k_p \mathbf{S}_t^{pq(1,2)} + k_q \mathbf{S}_c^{pq(1,2)} \\
 &\mathbf{S}^{pq(2,2)} = \mathbf{S}_c^{pq(2,2)}, \mathbf{S}^{pq(6,1)} = \mathbf{S}_t^{pq(6,1)}, \mathbf{S}^{pq(6,2)} = \mathbf{S}_c^{pq(6,2)} \\
 &\mathbf{S}^{pq(6,6)} = k_p \mathbf{S}_t^{pq(6,6)} + k_q \mathbf{S}_c^{pq(6,6)} \\
 &\text{If } \sigma^p < 0, \sigma^q > 0 : \mathbf{S}^{pq(1,1)} = \mathbf{S}_c^{pq(1,1)}, \mathbf{S}^{pq(1,2)} = k_p \mathbf{S}_c^{pq(1,2)} + k_q \mathbf{S}_t^{pq(1,2)} \\
 &\mathbf{S}^{pq(2,2)} = \mathbf{S}_t^{pq(2,2)}, \mathbf{S}^{pq(6,1)} = \mathbf{S}_c^{pq(6,1)}, \mathbf{S}^{pq(6,2)} = \mathbf{S}_t^{pq(6,2)} \\
 &\mathbf{S}^{pq(6,6)} = k_p \mathbf{S}_c^{pq(6,6)} + k_q \mathbf{S}_t^{pq(6,6)}
 \end{aligned} \quad (12)$$

Here, σ^p and σ^q denote the principal stresses. $\mathbf{S}_t^{pq}$ and $\mathbf{S}_c^{pq}$ are the tensile and compressive compliance matrices, respectively. k_p and k_q are the weight factors defined from the principal stresses:

$$k_p = \frac{|\sigma^p|}{|\sigma^p| + |\sigma^q|}, k_q = \frac{|\sigma^q|}{|\sigma^p| + |\sigma^q|} \quad (13)$$

For convenience, Eq. (12) can be rewritten in the following compact form:

$$\mathbf{S}_e^{pq} = \mathbf{W}_t(\sigma_e^p, \sigma_e^q) \circ \mathbf{S}_{t,e}^{pq} + \mathbf{W}_c(\sigma_e^p, \sigma_e^q) \circ \mathbf{S}_{c,e}^{pq} \quad (14)$$

where $\circ$ denotes the Hadamard product. The matrices $\mathbf{W}_t$ and $\mathbf{W}_c$ are stress-dependent weighting matrices defined by Eq. (12). Once the local tension/compression state has been identified in the current analysis, $\mathbf{W}_t$ and $\mathbf{W}_c$ are treated as fixed coefficient matrices in the subsequent sensitivity derivation. Accordingly, their implicit dependence on the design variables through the evolving stress state is neglected in the present work.

For anisotropic fiber composites, the optimized fiber direction in general does not completely coincide with the principal stress directions. As shown in Fig. 6, let α be the angle between the fiber and the principal stress directions. The tensile compliance matrix of fiber in the principal stress coordinate is obtained by rotating the tensile compliance matrix in the material frame:

$$\mathbf{S}_{t,e}^{pq} = \mathbf{T}(\alpha_e)^T \mathbf{S}_{t,e}^{\text{mat}} \mathbf{T}(\alpha_e) \quad (15)$$

where $\mathbf{T}(\cdot)$ is the rotation matrix and $\mathbf{S}_{t,e}^{\text{mat}}$ is the tensile compliance matrix in the material coordinates, which can be expressed by:

$$\mathbf{S}_{t,e}^{\text{mat}} = \begin{bmatrix} \frac{1}{E_{1,t,e}(d_e)} & \frac{-\nu_{12,t,e}(d_e)}{E_{1,t,e}(d_e)} & 0 \\ \frac{-\nu_{21,t,e}(d_e)}{E_{2,t,e}(d_e)} & \frac{1}{E_{2,t,e}(d_e)} & 0 \\ 0 & 0 & \frac{1}{G_{12,t,e}(d_e)} \end{bmatrix} \quad (16)$$

These elastic constants are previously given by Eq. (11). The compressive counterpart $\mathbf{S}_{c,e}^{\text{mat}}$ is obtained analogously from $\mathbf{S}_{t,e}^{\text{mat}}$. Substituting them into Eq. (14) yields the effective principal compliance matrix $\mathbf{S}_e^{pq}$. Finally, for purpose of global assembly, the compliance matrix is mapped to the global coordinate system by a rotation through angle β (between the principal stress direction and the global x-axis):

$$\mathbf{S}_e = \mathbf{T}(\beta_e)^T \mathbf{S}_e^{pq} \mathbf{T}(\beta_e) \quad (17)$$

In this work, the dual-field formulation is adopted. Let $\varphi_i, i \in \{1, 2\}$ denote the fiber orientation of field i in the global frame, and the misalignment between the fiber of field i and the principal stress direction is

$$\alpha_i = \varphi_i - \beta \quad (18)$$

Therefore, the tensile compliance matrix of field i in the material frame can be derived following Eq. (15):

$$\mathbf{S}_{t,i,e}^{pq} = \mathbf{T}(\alpha_{i,e})^T \mathbf{S}_{t,i,e}^{\text{mat}} \mathbf{T}(\alpha_{i,e}) \quad (19)$$

where the $\mathbf{S}_{t,i,e}^{\text{mat}}$ is parameterized by the gradient magnitude $d_{i,e}$ of geodesic field i . The compressive compliance matrix $\mathbf{S}_{c,i,e}^{pq}$ is obtained analogously. Applying the WCM weights yields the effective compliance matrix in the principal stress coordinates:

$$\mathbf{S}_{t,e}^{pq} = \mathbf{W}_t(\sigma_e^p, \sigma_e^q) \circ \mathbf{S}_{t,e}^{pq} + \mathbf{W}_c(\sigma_e^p, \sigma_e^q) \circ \mathbf{S}_{c,e}^{pq} \quad (20)$$

Finally, the two fields are mixed (e.g., by the phase weights W_1, W_2) to obtain the overall principal compliance matrix:

$$\mathbf{S}_e^{pq} = W_1 \mathbf{S}_{1,e}^{pq} + W_2 \mathbf{S}_{2,e}^{pq} \quad (21)$$

which is then rotated to the global system via Eq. (17).

For stiffness matrix assembly, the element compliance matrix is inverted to obtain the elastic constitutive matrix and coupled with SIMP

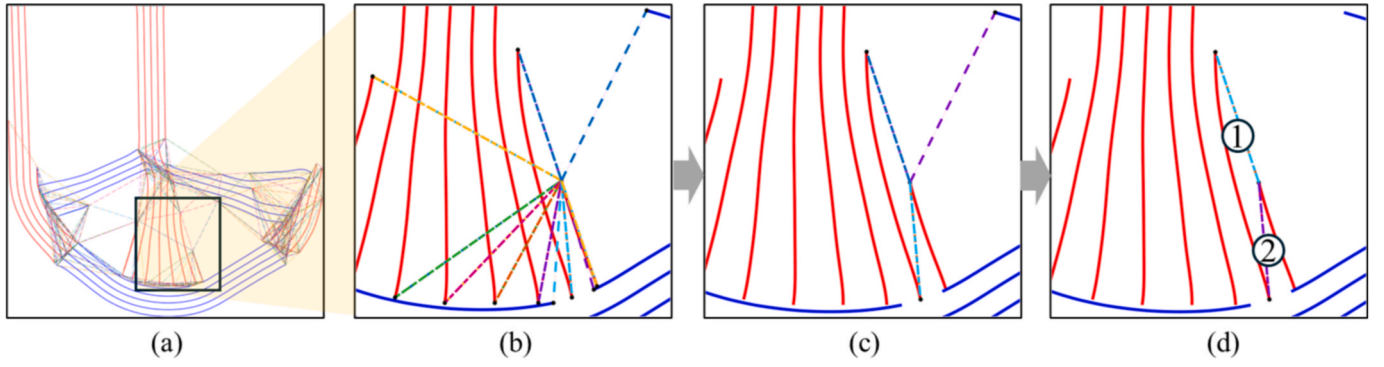


Fig. 9. Candidate bridges selection procedure. (a) Overview of the candidate bridges; (b) Zoom-in showing candidate bridges (dashed) generated between endpoint pairs within the distance threshold; (c) Removal of invalid candidates that intersect existing paths; (d) Removal of boundary-violating candidates and final selection by minimizing the including angle of paired endpoint tangential vectors.

density interpolation:

$$\mathbf{D}_e = \rho_e^p (\mathbf{S}_e)^{-1} \quad (22)$$

where p is the SIMP penalty. The element stiffness is finally obtained as:

$$\mathbf{k}_e = \int_{\Omega_e} \mathbf{B}^T \mathbf{D}_e \mathbf{B} d\Omega \quad (23)$$

with $\mathbf{B}$ the strain–displacement matrix. Assembling the elemental matrices $\mathbf{k}_e$ over all elements yields the global stiffness matrix $\mathbf{K}$, therefore completing the constitutive modeling of fiber composites within the finite-element formulation.

4.4. Post-processing

After the concurrent optimization, a five-step post-processing pipeline is proposed to transform the two-field trajectories into a globally feasible fiber path (see Fig. 7):

Step 1: Field alignment (see Fig. 7). The two optimized geodesic fields are aligned along the phase interface to smooth their junction. Let κ_1 and κ_2 denote the averages of ϕ_1 and ϕ_2 , respectively, on the portions of Γ where the fiber directions of the two phases are locally parallel (shown in Fig. 8(a)). The alignment uses

$$\phi_1 = \phi_1 + \frac{1}{2}(\kappa_1 - \kappa_2), \phi_2 = \phi_2 + \frac{1}{2}(\kappa_2 - \kappa_1) \quad (24)$$

The effect of this alignment is illustrated in Fig. 8(b–c). If the isocontours are extracted directly from the raw fields, they are noticeably misaligned across the interface. After alignment, the

isocontours show excellent match at the interface, which is beneficial for the subsequent path planning.

Step 2: Path extraction. Isocontours are extracted from the aligned fields with uniform spacing, by setting the step size h in Eq. (6) to unit length; consequently, fiber density is controlled solely by the optimized spacing variable s . After extraction, small line segments that arise near structural terminations are removed using a length threshold of 10 units, and the structural boundary contour is also extracted.

Step 3: Candidate bridges. For the remaining polylines, candidate bridges are generated between endpoint pairs whose separation is below a distance threshold of 30 units (see Fig. 9(a)). Candidates are partitioned into inter-phase and inner-phase sets.

Step 4: Connection strategy and constraints.

All bridges, whether inter-phase or inner-phase, satisfy the following constraints:

- no intersection with existing paths or previously added bridges (to avoid fiber overlap) (Fig. 9(c)).
- no excursion beyond the structural boundary (to prevent fibers from being exposed outside the structure) (Fig. 9(d)).

Valid candidates are then ranked by prioritizing bridges that minimize the including angle of the paired endpoint tangential vectors (thereby avoiding “Z-shaped” or overly sharp turns; see Fig. 9(d)). Final selection is determined by a greedy procedure executed in two stages:

- Inter-phase first. Bridges across the two phases are confirmed before same-phase connections. This prioritization reflects that fibers at the phase interface may be nearly continuous but separated by a minor gap, and connecting across the interface can enhance the continuity of fiber trajectories instead of generating extra truncations.

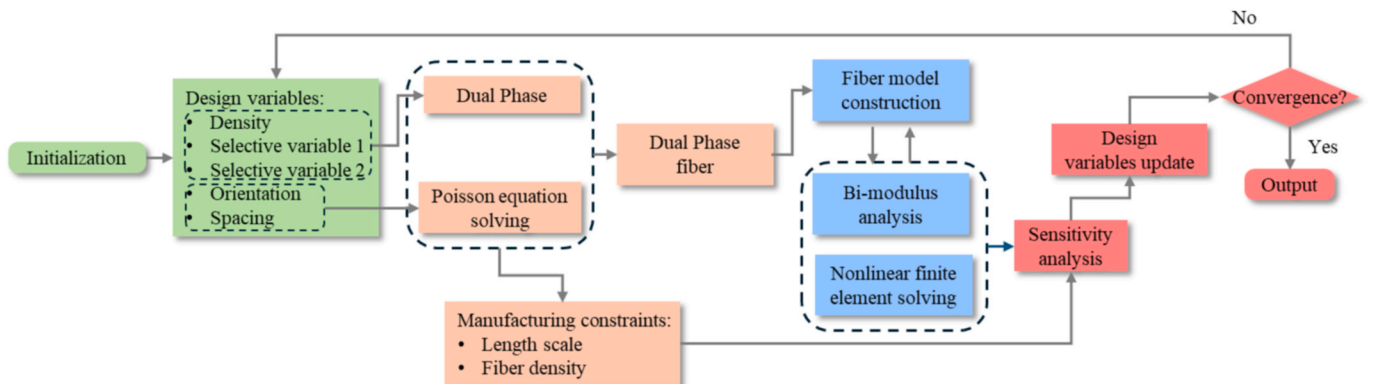


Fig. 10. Flowchart of the proposed method.

- (ii) Inner-phase next. After inter-phase connections, the remaining inner-phase endpoints are connected to promote overall continuity.

Step 5: Smoothing. The connected polylines are re-filtered to remove short paths using a 30-unit length threshold and then refined by B-spline smoothing to generate the final toolpaths. The resulting B-spline curves can be directly converted into G-code for printing.

By applying the above steps, fiber-path continuity and manufacturability are ensured with only minimal modification to the optimized design (the primary fiber directions and spacings remain unchanged).

5. Optimization model

To validate the effectiveness of the proposed framework, structural compliance (inverse stiffness) is used as the objective to benchmark performance in this work. Five design variables are optimized simultaneously: the density ρ , two selective variables x_1, x_2 , the angle θ , and the spacing variable s . The optimization model is:

$$\left\{ \begin{array}{l} \text{find : } \rho, x_1, x_2, \theta, s \\ \text{min : } c(\rho, x_1, x_2, \theta, s) \\ \text{st. } \mathbf{K}(\mathbf{U})\mathbf{U} = \mathbf{F} \\ V(\rho) \leq V_0 \\ d_{\max}(\theta, s) \leq d_0 \\ g_i^s = \frac{1}{N} \sum_{e=1}^N I_{i,e}^s \cdot [\min\{(\tilde{P}_{i,e} - \eta), 0\}]^2 \leq \varepsilon \\ g_i^v = \frac{1}{N} \sum_{e=1}^N I_{i,e}^v \cdot [\min\{((1 - \eta) - \tilde{P}_{i,e}), 0\}]^2 \leq \varepsilon, i = 1, 2. \\ \rho_e, x_{1,e}, x_{2,e}, s_e \in [0, 1] \\ \theta_e \in \left[-\frac{\pi}{4}, \frac{\pi}{4}\right]. \end{array} \right. \quad (25)$$

Here, c denotes the structural compliance. The equilibrium equation involves the global stiffness matrix $\mathbf{K}(\cdot)$, the displacement vector $\mathbf{U}$, and the external load vector $\mathbf{F}$. Because the constitutive response incorporates tension-compression (bi-modulus) asymmetry, $\mathbf{K}$ depends on the stress state and hence on $\mathbf{U}$, and therefore, the finite-element system is nonlinear and solved by Newton iterations [56].

$V(\rho)$ is the total material volume with an upper bound V_0 . The functions g^s and g^v are manufacturing length-scale constraints introduced to prevent undersized features in the phase partition. The corresponding indicators I^s and I^v are defined as:

$$I_i^s = \bar{P}_i \cdot \exp\left(-\zeta \cdot |\nabla \tilde{P}_i|^2\right), I_i^v = (1 - \bar{P}_i) \cdot \exp\left(-\zeta \cdot |\nabla \tilde{P}_i|^2\right) \quad (26)$$

where P_i denotes the two-phase field, as defined in Eq. (2). Here, $\bar{P}$ and $\tilde{P}_i$ represent the Heaviside-projected field and the filtered field of P_i , respectively. The parameter ζ controls the skeleton extraction and is set to 10^3 . The parameter $\eta \in [0.5, 1]$ controls the size threshold enforced by the manufacturing constraints, with larger values of η leading to thicker structural features. In the present work, $\eta = 0.7$. In addition, a small numerical constant ε is introduced for numerical stability, with $\varepsilon = 10^{-6}$. A more detailed description of this constraint can be found in Ref. [65].

Fig. 10 illustrates the flowchart of the algorithm procedures, which consists of four main steps:

Step 1: Initialize the parameters and five sets of design variables.

Step 2: The density field ρ is coupled with the selective variables (x_1, x_2) to form two phases. The orientation θ and spacing s are used to generate two orthogonal geodesic fields (via a Poisson problem), which determine the fiber paths. Manufacturing constraints are imposed, including the length scale constraints and the maximum fiber density constraint.

Step 3: Given the current structure and fiber paths, the composite is homogenized and its tension-compression asymmetry is incorporated. The resulting nonlinear finite-element system is solved by Newton iterations, updating the effective material response until convergence at the current design iteration.

Step 4: Sensitivities of the objective and all constraints are evaluated, and the design variables are updated using the method of moving asymptotes (MMA) [66]. If the design satisfies the convergence criteria, the optimization terminates; otherwise, Steps 2–4 are repeated.

6. Sensitivity analysis

This section outlines the first-order derivatives required by the MMA optimizer. The sensitivities of standard linear terms, such as the volume constraint, are well established in Ref. [67] and are omitted here for brevity. The sensitivity of compliance with respect to the selection variables follows a similar pattern and is summarized below:

6.1. Sensitivity w.r.t. selective variable $x_{1,e}$

$$\frac{\partial c}{\partial x_{1,e}} = - \sum_{e=1}^N (\rho_e)^p \mathbf{u}_e^T \frac{\partial \mathbf{k}_e}{\partial x_{1,e}} \mathbf{u}_e \quad (27)$$

$\mathbf{u}_e$ denotes the element nodal displacement vector and $\mathbf{k}_e$ the element stiffness matrix. By the chain rule,

$$\frac{\partial \mathbf{k}_e}{\partial x_{1,e}} = \frac{\partial \mathbf{k}_e}{\partial \mathbf{S}_j} \frac{\partial \mathbf{S}_j}{\partial x_{1,e}} \quad (28)$$

where $\frac{\partial \mathbf{k}_e}{\partial \mathbf{S}_j}$ is given in Ref. [56], and $\frac{\partial \mathbf{S}_j}{\partial x_{1,e}}$ follows from the frame transforms:

$$\frac{\partial \mathbf{S}_j}{\partial x_{1,e}} = \mathbf{T}(\beta_j)^T \frac{\partial \mathbf{S}_j^{\text{pq}}}{\partial x_{1,e}} \mathbf{T}(\beta_j) \quad (29)$$

$\frac{\partial \mathbf{S}_j^{\text{pq}}}{\partial x_{1,e}}$ can be derived from Eq. (21),

$$\frac{\partial \mathbf{S}_j^{\text{pq}}}{\partial x_{1,e}} = \frac{\partial W_{1,j}}{\partial x_{1,e}} \mathbf{S}_{1,j}^{\text{pq}} + \frac{\partial W_{2,j}}{\partial x_{1,e}} \mathbf{S}_{2,j}^{\text{pq}} \quad (30)$$

where the weight sensitivities $\frac{\partial W_{ij}}{\partial x_{1,e}}$ follow directly from Eq. (1)

$$\frac{\partial W_{1,j}}{\partial x_{1,e}} = \delta_{j,e} \frac{\partial W_{1,e}}{\partial x_{1,e}} = \frac{(\omega_{1,e} + \omega_{2,e}) \frac{\partial \omega_{1,e}}{\partial x_{1,e}} - \omega_{1,e} \left(\frac{\partial \omega_{1,e}}{\partial x_{1,e}} + \frac{\partial \omega_{2,e}}{\partial x_{1,e}} \right)}{(\omega_{1,e} + \omega_{2,e})^2} \quad (31)$$

Here $\delta_{j,e}$ denotes the Kronecker delta ($\delta_{j,e} = 1$ if $j = e$, and 0 otherwise). An analogous relation holds for $\frac{\partial W_{2,j}}{\partial x_{1,e}}$. Then, $\frac{\partial \omega_{1,e}}{\partial x_{1,e}}$ can be expressed by:

$$\frac{\partial \omega_{1,e}}{\partial x_{1,e}} = q x_{1,e}^{(q-1)} (1 - x_{2,e}^q) \quad (32)$$

and analogously for $\frac{\partial \omega_{2,e}}{\partial x_{1,e}}$.

Table 1

Constituent parameter sets for the interpolation constitutive model [53].

Parameter set	$E_{1,t}$	$E_{2,t}$	$\nu_{12,t}$	$G_{12,t}$	$E_{1,c}$	$E_{2,c}$	$\nu_{12,c}$	$G_{12,c}$
f.1	69.4	3.5	0.33	1.9	69.4	3.5	0.33	1.9
f.2	69.4	3.5	0.33	1.9	34.7	3.5	0.33	1.9
f.3	69.4	3.5	0.33	1.9	6.94	3.5	0.33	1.9
m	4.76	4.76	0.37	1.74	4.76	4.76	0.37	1.74

6.2. Sensitivity w.r.t. orientation θ_e

The same derivation process as in Eq. (27)–(29) applies; Then,

$$\frac{\partial \mathbf{S}_f^{pq}}{\partial \theta_e} = \mathbf{W}_{1,j} \frac{\partial \mathbf{S}_{1,j}^{pq}}{\partial \theta_e} + \mathbf{W}_{2,j} \frac{\partial \mathbf{S}_{2,j}^{pq}}{\partial \theta_e} \quad (33)$$

The WCM combination gives:

$$\frac{\partial \mathbf{S}_{t,j}^{pq}}{\partial \theta_e} = \mathbf{W}_t(\sigma_e^p, \sigma_e^q) \circ \left(\frac{\partial \mathbf{S}_{t,j}^{pq}}{\partial \theta_e} \right) + \mathbf{W}_c(\sigma_e^p, \sigma_e^q) \circ \left(\frac{\partial \mathbf{S}_{c,j}^{pq}}{\partial \theta_e} \right) \quad (34)$$

Each term, e.g. the tensile part, satisfies

$$\frac{\partial \mathbf{S}_{t,j}^{pq}}{\partial \theta_e} = \frac{\partial \mathbf{T}(\alpha_{ij})}{\partial \theta_e} \mathbf{S}_{t,j}^{\text{mat}} \mathbf{T}(\alpha_{ij}) + \mathbf{T}(\alpha_{ij})^T \frac{\partial \mathbf{S}_{t,j}^{\text{mat}}}{\partial \theta_e} \mathbf{T}(\alpha_{ij}) + \mathbf{T}(\alpha_{ij})^T \mathbf{S}_{t,j}^{\text{mat}} \frac{\partial \mathbf{T}(\alpha_{ij})}{\partial \theta_e} \quad (35)$$

Since $\alpha_{ij} = \phi_{ij} - \beta_j$, one has:

$$\frac{\partial \mathbf{T}(\alpha_{ij})}{\partial \theta_e} = \frac{\partial \mathbf{T}(\alpha_{ij})}{\partial \alpha_{ij}} \frac{\partial \alpha_{ij}}{\partial \theta_e}, \quad \frac{\partial \alpha_{ij}}{\partial \theta_e} = \frac{\partial \phi_{ij}}{\partial \theta_e} \quad (36)$$

$\frac{\partial \phi_{ij}}{\partial \theta_e}$ is expressed by

$$\frac{\partial \phi_{ij}}{\partial \theta_e} = \frac{\partial \text{atan } 2 \left(-\frac{\partial \phi_{ij}}{\partial x}, \frac{\partial \phi_{ij}}{\partial y} \right)}{\partial \theta_e} = \frac{\nabla^\perp \phi_{ij} \cdot \nabla \left(\frac{\partial \phi_{ij}}{\partial \theta_e} \right)}{\|\nabla \phi_{ij}\|^2} \quad (37)$$

where $\nabla^\perp \phi_{ij} = \left(-\frac{\partial \phi_{ij}}{\partial y}, \frac{\partial \phi_{ij}}{\partial x} \right)$ (a 90° rotation of $\nabla \phi$). $\frac{\partial \phi_{ij}}{\partial \theta_e}$ follows by chain rule through the field source location $\mathbf{X}_{i,e}$:

$$\frac{\partial \phi_{ij}}{\partial \theta_e} = \frac{\partial \phi_{ij}}{\partial \mathbf{X}_{i,e}} \frac{\partial \mathbf{X}_{i,e}}{\partial \theta_e}, \quad (38)$$

where $\frac{\partial \phi_{ij}}{\partial \mathbf{X}_{i,e}}$ can refer to Refs. [13,57], $\frac{\partial \mathbf{X}_{i,e}}{\partial \theta_e}$ can be expressed by:

$$\frac{\partial \mathbf{X}_{1,e}}{\partial \theta_e} = s_e \begin{bmatrix} -\sin(\theta_e) \\ \cos(\theta_e) \end{bmatrix}, \quad \frac{\partial \mathbf{X}_{2,e}}{\partial \theta_e} = -s_e \begin{bmatrix} \cos(\theta_e) \\ \sin(\theta_e) \end{bmatrix} \quad (39)$$

Another derivative in Eq. (35) $\frac{\partial \mathbf{S}_{t,j}^{\text{mat}}}{\partial \theta_e}$ can be obtained by chain rule:

$$\frac{\partial \mathbf{S}_{t,j}^{\text{mat}}}{\partial \theta_e} = \frac{\partial \mathbf{S}_{t,j}^{\text{mat}}}{\partial d_{i,j}} \frac{\partial d_{i,j}}{\partial \theta_e} \quad (40)$$

$\frac{\partial \mathbf{S}_{t,j}^{\text{mat}}}{\partial d_{i,j}}$ can be derived by Eq. (11). $\frac{\partial d_{i,j}}{\partial \theta_e}$ can be calculated from:

$$\frac{\partial d_{i,j}}{\partial \theta_e} = \frac{\partial \|\nabla \phi_{ij}\|}{\partial \theta_e} = \frac{\nabla \phi_{ij} \cdot \nabla \left(\frac{\partial \phi_{ij}}{\partial \theta_e} \right)}{\|\nabla \phi_{ij}\|} \quad (41)$$

$\frac{\partial \phi_{ij}}{\partial \theta_e}$ has been given in Eq. (38).

6.3. Sensitivity w.r.t. spacing s_e

Analogously,

$$\frac{\partial \mathbf{S}_f^{pq}}{\partial s_e} = \mathbf{W}_{1,j} \frac{\partial \mathbf{S}_{1,j}^{pq}}{\partial s_e} + \mathbf{W}_{2,j} \frac{\partial \mathbf{S}_{2,j}^{pq}}{\partial s_e} \quad (42)$$

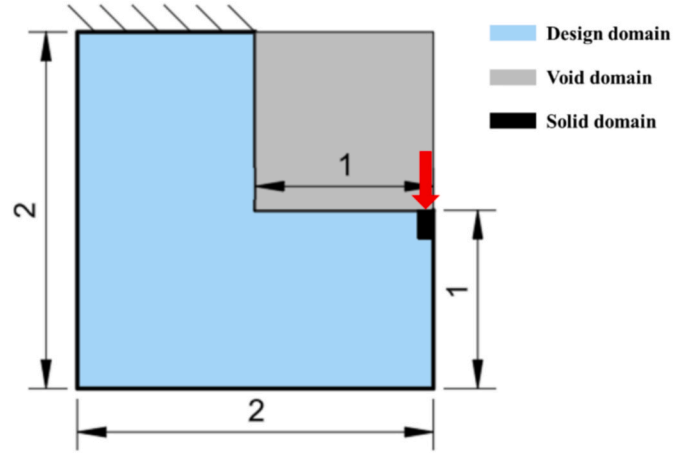


Fig. 11. Boundary conditions of L-bracket.

with the same derivative process as Eq. (33)–(39) but using:

$$\frac{\partial \mathbf{X}_{1,e}}{\partial s_e} = \begin{bmatrix} \cos(\theta_e) \\ \sin(\theta_e) \end{bmatrix}, \quad \frac{\partial \mathbf{X}_{2,e}}{\partial s_e} = \begin{bmatrix} -\sin(\theta_e) \\ \cos(\theta_e) \end{bmatrix} \quad (43)$$

6.4. Sensitivity of the fiber density constraint w.r.t. θ_e & s_e

With the p -norm approximation

$$\frac{\partial d_{\max}}{\partial \theta_e} = \left(\sum_{i=1}^2 \sum_{e=1}^N (d_{i,e})^\mu \right)^{\frac{1}{\mu}-1} \sum_{e=1}^N (d_{i,e})^{\mu-1} \frac{\partial d_{i,e}}{\partial \theta_e} \quad (44)$$

where $\frac{\partial d_{i,e}}{\partial \theta_e}$ has been calculated in Eq. (41). $\frac{\partial d_{\max}}{\partial s_e}$ is derived from the same process. So far, all the key sensitivities have been provided.

7. Numerical examples

This section presents numerical examples to demonstrate the effectiveness of the proposed model. All cases are executed on a 12-core desktop workstation (Intel® Xeon® W-2195, 2.30 GHz). All algorithms are implemented in MATLAB. Unless otherwise stated, the initialization is as follows: the density field is set uniformly to the prescribed maximum volume fraction, the selective variables are initialized uniformly to be 0.5, the orientation variable is set to 0, and the spacing variable is initialized to half of the maximum admissible fiber density. In SIMP, the penalty parameter is $p = 3$. Given DMO interpolation, the penalization parameter q is ramped from 3, increasing by 1 every 10 iterations until it reaches 11. The optimization process terminates when the compliance changes by less than 10^{-2} while all constraints are satisfied, or after a maximum of 110 iterations.

The constituent parameter sets used in Eq. (11) are summarized in Table 1. The parameter sets f.1, f.2 and f.3 correspond to three different bi-modulus ratios. Under the f.1 setting, the interpolation model exhibits no tension-compression asymmetry in the elastic moduli. From f.2 to f.3, the asymmetry between the tensile and compressive moduli becomes increasingly pronounced. These three parameter settings allow a systematic investigation of how different degrees of bi-modulus behavior

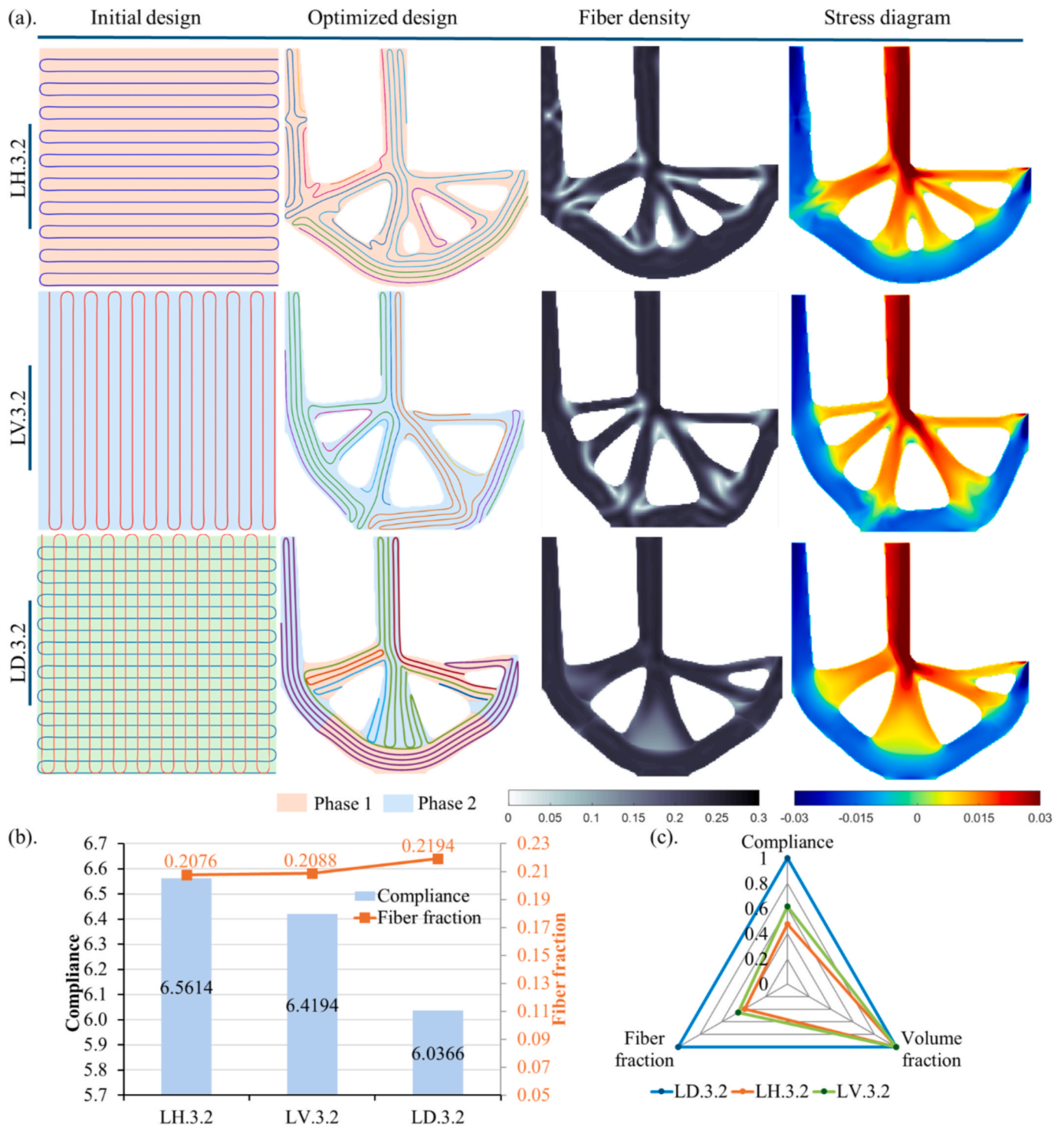


Fig. 12. Single-vs. dual-field fiber-path optimization on the L-bracket. (a) Initial fields and resulting design; (b) Comparison of compliance (bars) and fiber fraction (line); (c) Radar plot.

affect the optimized structural layouts and the resulting fiber-path distributions. Dimensionless material settings are adopted throughout numerical examples.

Throughout, cases are named by the pattern R M. O. Q, where R encodes the boundary condition (e.g., L for the L-bracket), M indicates the method (H for single-field with a horizontal initial fiber orientation, V for single-field with a vertical initial fiber orientation, and D for dual-field), O denotes the maximum fiber density, and Q specifies the parameter set listed in Table 1. For example, LD.3.2 designates the L-

bracket under the dual-field method with a maximum fiber density of 0.3 using f.2 from Table 1.

7.1. L-bracket

The first numerical example is an L-bracket. The boundary conditions are shown in Fig. 11. The design domain is a square plate with an aspect ratio of 1:1, partitioned into design and non-design regions. The non-design regions include: a passive void domain is located at the top-

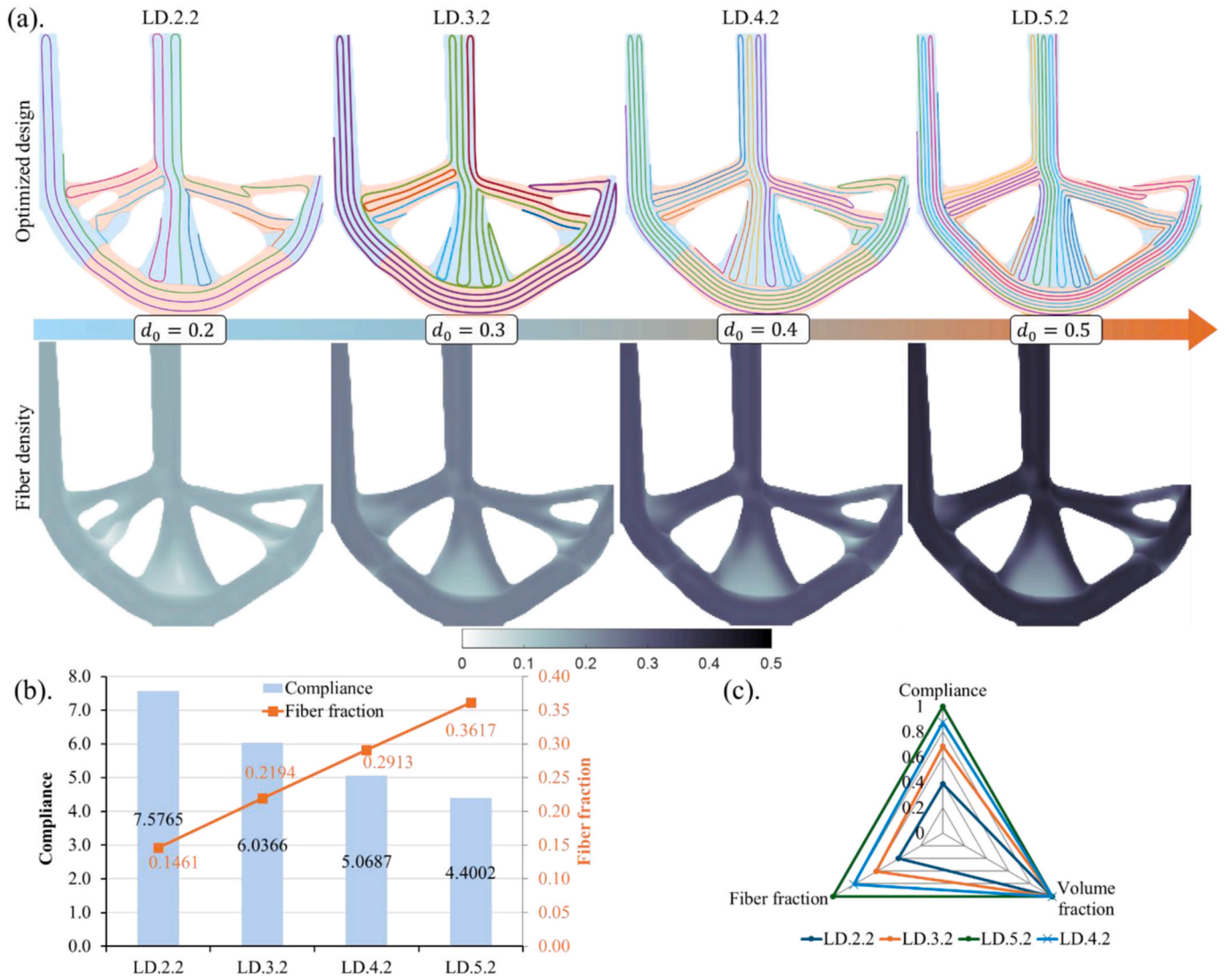


Fig. 13. Comparison of different fiber densities. (a) Optimized designs for different maximum fiber density setting ($d_0 = 0.2, 0.3, 0.4, 0.5$); (b) Comparison of compliance (bars) and fiber fraction (line); (c) Radar plot.

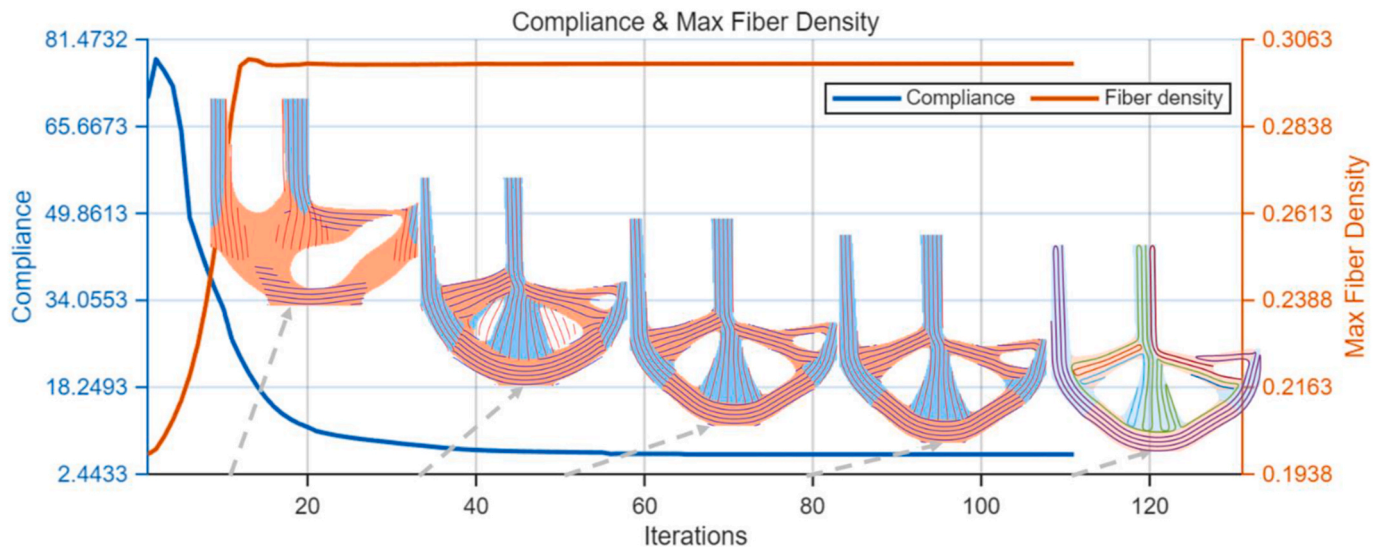


Fig. 14. Iteration history.

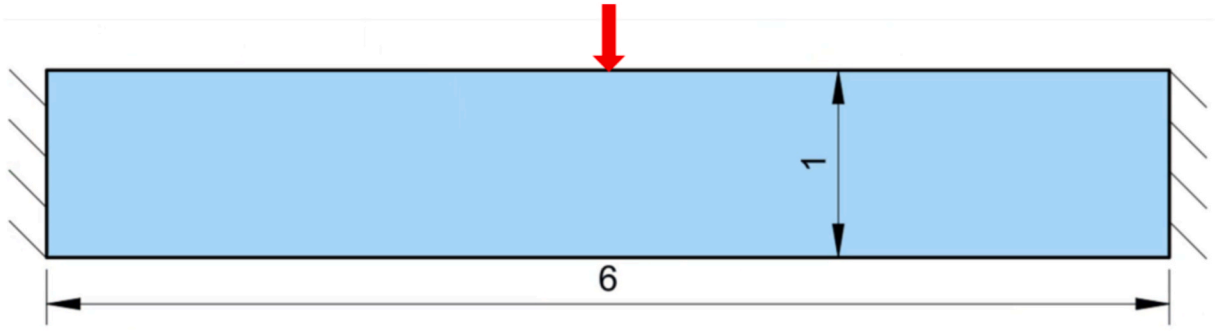


Fig. 15. Boundary conditions of double-clamped beam.

right corner, with its side length as one half of the overall domain edge length; a passive solid domain is defined at the center of the right edge, with a width of one element and a height of five elements; this domain serves as the load-bearing region, where a unit force is applied. The top boundary of the plate is fixed. For finite-element analysis, the domain is discretized into a 200×200 mesh with square elements. The maximum volume fraction is set to 0.5, and the maximum fiber density d_0 is 0.3.

7.1.1. Single phase vs. dual phase

This subsection compares the results obtained using the single-field formulation and the dual-field formulation.

As shown in Fig. 12, under the single-field optimization, the result is highly sensitive to the initialization of the fiber-orientation field: setting the initial direction to vertical (LV.3.2) versus horizontal (LH.3.2) leads to markedly different fiber path layouts and local densities. In addition, the fiber distribution is non-uniform, fiber density is typically low near member junctions. The stress diagram displays the signed principal stress. Positive values (red) indicate tensile principal stress σ^p and negative values (blue) indicate compressive principal stress σ^q . Besides the stress concentrations induced purely by the structural geometry,

additional hotspots are observed in regions where fiber paths meet with nearly opposite directions, resulting in conflicting fiber orientations and elevated stresses.

In contrast, because the initial design uses two orthogonal orientation fields, the dual-field scheme eliminates the need to decide whether the initial directions should be vertical or horizontal. During optimization, the two fields produce interleaved path families whose fills do not overlap, yielding uniform spacing and dense layout of the fiber trajectories by weakening the local drops in fiber density at junctions.

The bar/line chart shows that, relative to the single-field baselines, the dual-field solution achieves the lowest compliance, $c = 6.5614$ (LH.3.2), 6.4194 (LV.3.2), and 6.036 (LD.3.2), while maintaining the highest fiber fraction under the same maximum density cap. The total fiber fraction η is computed as:

$$\eta = \frac{\sum_{e=1}^N d_e}{VN} \quad (45)$$

that yields $\eta = 0.2076$ (LH.3.2), 0.2088 (LV.3.2), and 0.2194 (LD.3.2). The radar plot, which consolidates multiple criteria (e.g., compliance,

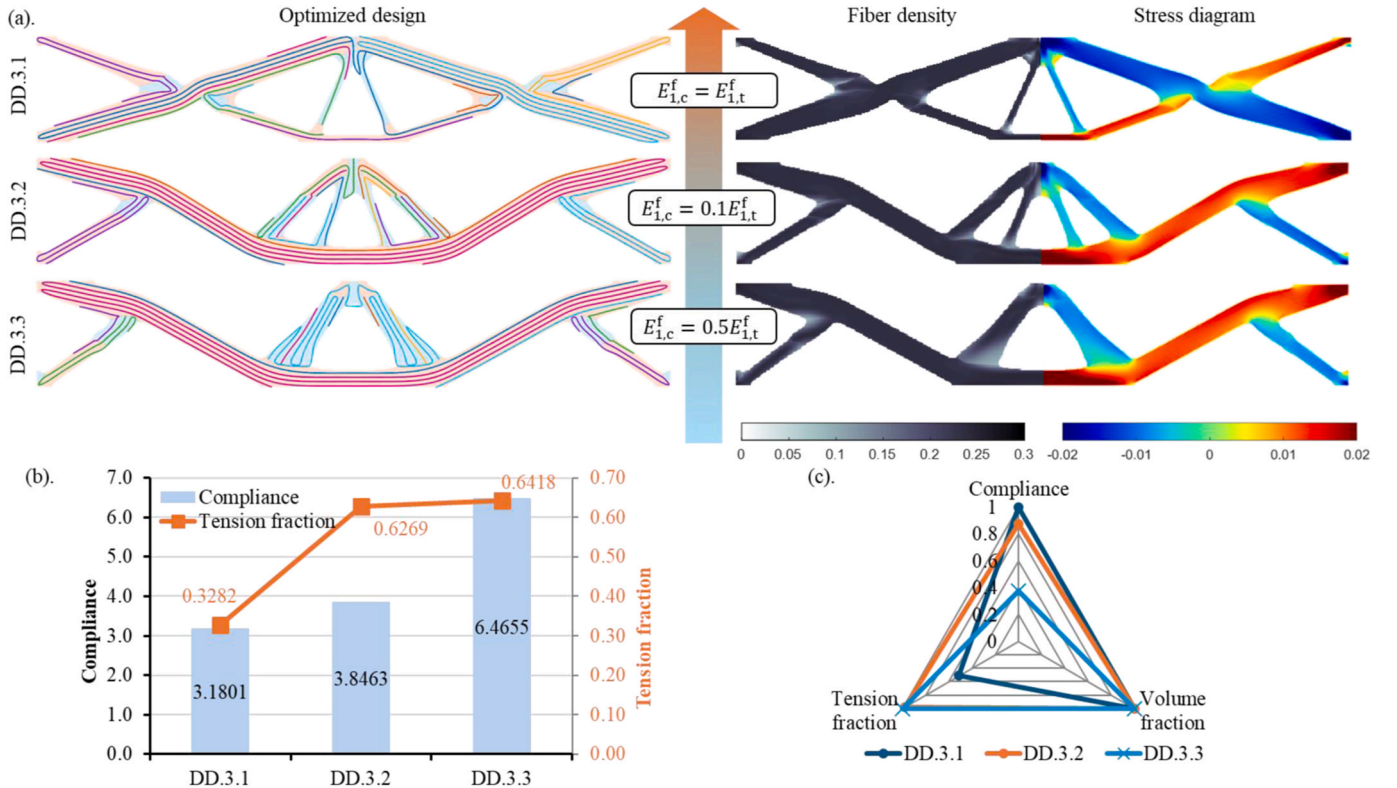


Fig. 16. Comparison of different bi-modulus ratios. (a) Optimized designs; (b) Comparison of compliance (bars) and tension fraction (lines); (c) Radar plot.

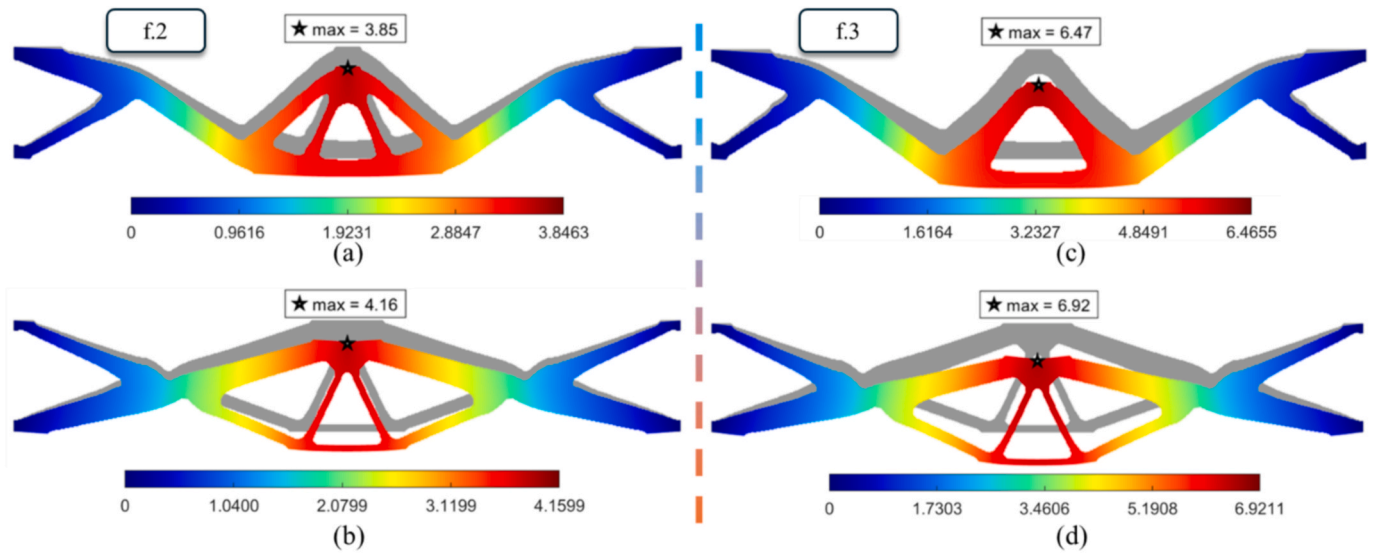


Fig. 17. Comparative analysis with bi-modulus modeling. (a) DD.3.2 with f.2, (b) DD.3.1 with f.2, (c) DD.3.2 with f.3, and (d) DD.3.1 with f.3.

fiber fraction), indicates consistent improvements for the dual-field setting. Overall, the dual-field formulation produces a denser and more uniform fiber distribution with better mechanical performance and stronger robustness to initialization, making it preferable for fabrication-oriented design.

7.1.2. Comparison of different fiber densities

This subsection examines the influence of the maximum fiber density d_0 by comparing $d_0 = 0.2, 0.3, 0.4, 0.5$. As shown in Fig. 13(a), when $d_0 = 0.2$ the limited fiber budget cannot sustain multiple load-carrying branches, fiber is concentrated along the primary load path and secondary branches are weakened. Consequently, the second branch undergoes a topological reconfiguration (appears a new hole).

Once $d_0 \geq 0.3$, the design crosses a topological stability threshold: the major load paths and holes remain essentially unchanged, and additional fiber is used mainly to densify existing paths rather than create new ones. Consistently, the global fiber fraction increases from about 0.2194 to 0.2913 and 0.3617 as d_0 rises from 0.3 to 0.4 and 0.5, while the compliance decreases from roughly 6.04 to 5.07 and 4.40, respectively. Across all cases, regions of higher fiber density concentrate near the L-bracket corner (the typical stress-concentration zone), whereas low-stress regions remain more sparsely filled, indicating an expected and physically reasonable distribution.

7.1.3. Iteration history

Fig. 14 traces the evolution of one of the designs (LD.3.2). The

compliance (left) exhibits a rapid drop during the early iterations, followed by a steady, monotonic decline toward a platform. In parallel, the maximum fiber density (right) rises quickly to the prescribed cap and then remains essentially constant thereafter. The insets show representative layouts at selected iterations: the overall topology stabilizes early, while subsequent updates primarily refine the fiber paths. The convergence curves are smooth, with only minor oscillations.

7.2. Double clamped beam

The second example, based on the double-clamped structure, examines how tension–compression asymmetry (bi-modulus behavior) influences both the structural topology and the fiber-path design.

The boundary conditions of the double-clamped beam are shown in Fig. 15. The domain is a longitudinal beam with an aspect ratio of 6:1. The left and right edges are clamped, and a unit vertical load is applied at the midpoint of the top edge. The design domain is discretized into a 600×100 mesh of square finite elements. The maximum volume fraction is set to 0.35, and the maximum fiber density d_0 is 0.3.

7.2.1. Comparison of different bi-modulus ratios

Fig. 16 compares optimization outcomes for the three bi-modulus ratios in Table 1 and shows a clear, monotonic trend as the compressive modulus decreases. With $E_{1,c}^f = E_{1,t}^f$, compressive members are utilized extensively, and the path layout is nearly symmetric between

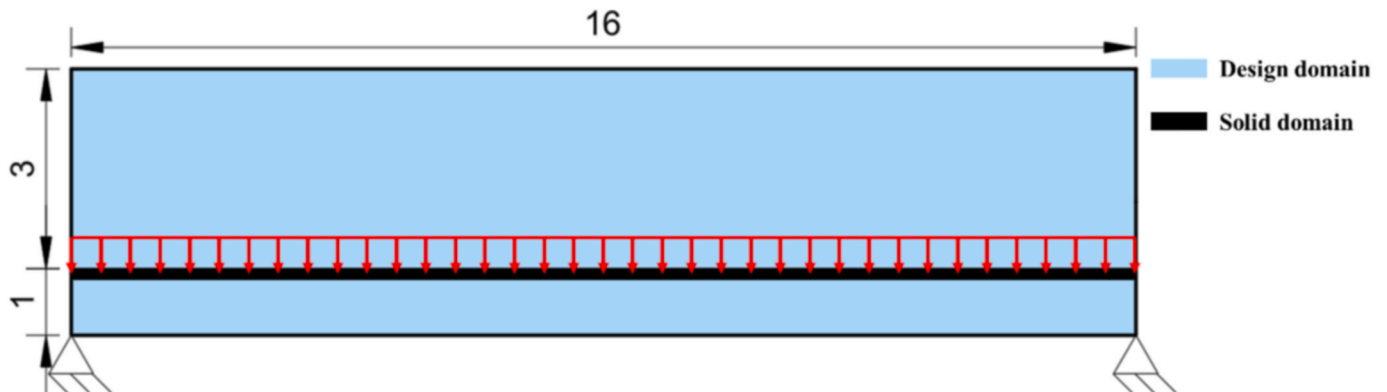


Fig. 18. Bridge-shaped structure.

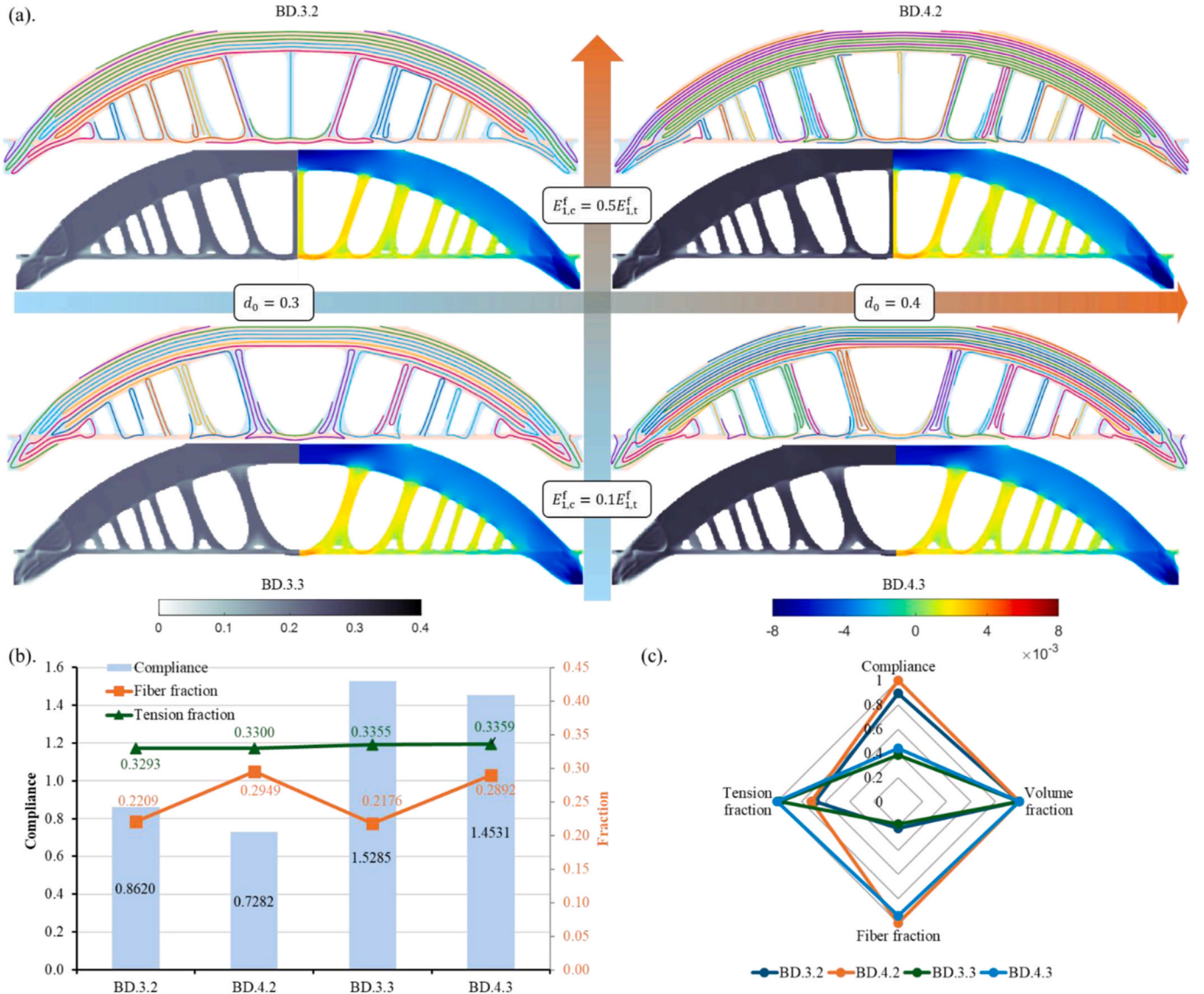


Fig. 19. Bridge structures. (a) Optimized designs for different max fiber densities and bi-modulus ratios; (b) Comparison of compliance (bars) and fiber fraction (lines); (c) Radar plot.

tension and compression. Reducing $E_{1,c}^f$ to $0.5E_{1,t}^f$ induces a reorientation toward tension-dominant trajectories: compressed branches become slimmer and shorter, whereas tensile ties are reinforced and lengthened. At $E_{1,c}^f = 0.1E_{1,t}^f$, compressive load paths are largely suppressed, yielding a more truss-like, tension-based architecture and shifting the stress field into tension near the mid-span and at the supports.

This progression is expected from bi-modulus behavior: lowering $E_{1,c}^f$ diminishes the composite's capacity to carry compressive load, so the optimizer penalizes the formation of compression-dominated regions and preferentially allocates material to tension paths. The result is a systematic evolution from tension-compression symmetry to a tensile load-bearing network as E_c/E_t decreases.

The quantitative trends are consistent with the above observations. Compliance increases as E_c/E_t decreases, approximately 3.18, 3.85 and 6.47, reflecting the loss of global stiffness when compression is penalized. Meanwhile, the tension-dominant fraction of the domain (from the WCM classification) grows from 0.33 to 0.63 and 0.64, indicating that the optimizer progressively generates more tensile members as compression becomes less effective; the increase saturates between the moderate and strong asymmetry cases. The radar chart compares the

three optimized designs under a common fiber density cap $d_0 = 0.3$. Because the same cap is used, the optimized fiber fraction converges to essentially the same value (0.2241) for all three cases. Note that this value falls below the cap due to the approximated p-norm aggregation to enforce the maximum fiber density constraint.

Overall, bi-modulus anisotropy directly shapes the optimal fiber architecture. A moderate asymmetry already captures the tension-preferred behavior and yields balanced paths with limited compliance penalty, whereas an excessively low E_c further suppresses compression but at the cost of a substantial stiffness loss. These results provide a practical guideline for fabrication-oriented design.

7.2.2. With vs. without bi-modulus: comparative analysis

This section compares designs optimized with bi-modulus behavior against designs optimized without it to highlight the importance of modeling tension-compression asymmetry during optimization.

As shown in Fig. 17, panel (a) reports the displacement field of DD.3.2, a design optimized with bi-modulus behavior and evaluated using parameter set f.2, yielding a peak displacement of 3.85 at the load application point. For a fair comparison, panel (b) shows DD.3.1, which was optimized assuming equal tensile and compressive moduli, but is re-

Table 2

Specimen dimensions.

Load type	Load direction	Standard specimen size (mm)	Reference standard
Tensile test	Longitudinal	240 × 12.5 × 2	GB/T 3354-2014
	Transverse	180 × 25 × 3	
Compressive test	Longitudinal	52 × 12 × 12	GB/T 1448-2005
	Transverse	52 × 12 × 12	
Shear test	In-plane shear	250 × 25 × 1.2	GB/T 3355-2014

evaluated using parameter set f.2 while keeping its topology and fiber paths unchanged; its maximum displacement is 4.16, an 8% increase relative to (a), indicating lower stiffness under identical fiber properties. The same trend holds for parameter set f.3: panel (c) (DD.3.3 with f.3) attains a peak displacement of 6.47, whereas panel (d) (DD.3.1 re-evaluated with f.3) reaches 6.92, a 7% increase.

Takeaway. Designs optimized without tension–compression asymmetry modeling but evaluated with bi-modulus properties exhibit noticeably inferior stiffness. Incorporating bi-modulus effects during optimization is therefore essential to obtain architectures that fully leverage the composite material behavior.

7.3. Bridge-shaped structure

The final case is a bridge-shaped structure (boundary conditions shown in Fig. 18). The design domain has an aspect ratio of 1:4 and is discretized into 160 × 640 square elements. The bottom-left and bottom-right corners are clamped. A solid domain of 5 elements in height is prescribed at $y = 1/4$ (measured from the bottom) and serves as a deck; a uniform unit linear load is applied on the upper face of this deck. The maximum volume fraction is set to 0.35.

Fig. 19 compares optimized designs with both varying fiber-density cap and the bi-modulus ratio. Across all cases the macro topology is stable: an arched upper chord and a set of inclined web ribs carry the load from mid-span toward the supports. The dual-phase paths remain continuous and interleaved, providing uniform coverage along the arch

and within the webs.

Increasing d_0 primarily densifies the fiber network, while the global topology remains essentially unchanged. The bar/line chart confirms a higher fiber fraction and lower compliance as d_0 increases: for $E_{1,c}^f = 0.5E_{1,t}^f$, compliance decreases from 1.5285 to 1.4531 as the fiber fraction rises from 0.2176 to 0.2892; for $E_{1,c}^f = 0.1E_{1,t}^f$, compliance decreases from 0.8620 to 0.7282 as the fiber fraction rises from 0.2209 to 0.2949.

Varying the compressive modulus does not cause a drastic switch between tension- and compression-dominated behavior. For both $E_{1,c}^f = 0.5E_{1,t}^f$ and $E_{1,c}^f = 0.1E_{1,t}^f$, the structure still behaves as a compression arch with tensioned ribs and deck. As $E_{1,c}^f$ is reduced, the slender rib at the middle vanishes and the neighboring ribs become thicker. However, the tensile-stress volume fraction increases only slightly: 0.3293 to 0.3300 for $E_{1,c}^f = 0.5E_{1,t}^f$ (BD.3.2 and BD.4.2), 0.3355 to 0.3359 for $E_{1,c}^f = 0.1E_{1,t}^f$ (BD.3.3 and BD.4.3), respectively. This indicates that the global balance between tension and compression remains similar despite these topological rearrangements.

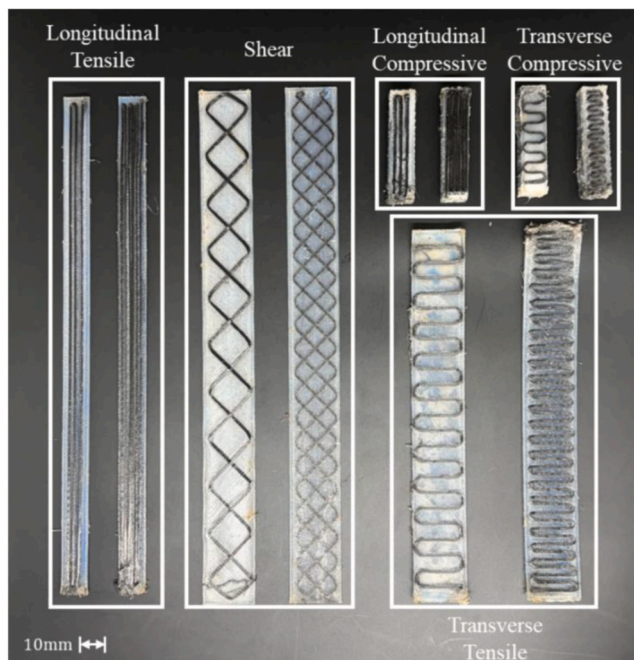
8. Experimental validation

To verify the effectiveness of the proposed method, experimental validation was carried out by fabricating specimens using the self-developed continuous-fiber 3D printing system (Fibresin) at Shandong University, as described in Fig. 22 (a) and [68–70]. The experimental study consisted of two main steps. First, material characterization tests were conducted to establish the interpolated constitutive model of the

Table 3

Measured moduli (GPa).

Fiber volume fraction	Longitudinal Tensile	Longitudinal Compressive	Transverse Tensile	Transverse Compressive	Shear
20%	13.38	2.81	2.64	1.11	1.69
40%	23.41	3.92	2.14	0.90	1.42



(a). Printed specimen



(b). Tested specimen

Fig. 20. Specimens for constitutive model characterization: (a) printed specimens; (b) tested specimens.

Table 4
Identified constituent parameter values in the interpolated model of Eq. (11).

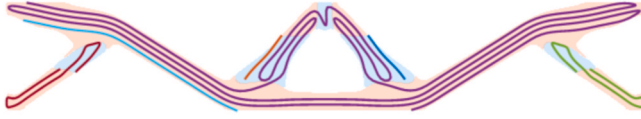
Parameter set	$E_{1,t}$	$E_{2,t}$	$\nu_{12,t}$	$G_{12,t}$	$E_{1,c}$	$E_{2,c}$	$\nu_{12,c}$	$G_{12,c}$
f	53.50	1.36	0.33	0.96	7.25	0.57	0.33	0.96
m	3.35	3.45	0.37	2.1	1.70	1.45	0.37	2.1

CFRCs. Subsequently, based on the identified constitutive parameters, a representative case was optimized, and the resulting design was validated through specimen fabrication and mechanical testing.

8.1. Constitutive model

The matrix material used in the experiments was polyamide (PA) filament (ePA, eSUN), whereas the reinforcement material was continuous carbon fiber-reinforced polyamide composite (CCF/PA, Mark-forged). Following the testing methodology reported by Hou et al. [55], specimens with two different fiber volume fractions, 20% and 40%, were fabricated for material characterization. The tests included five loading cases: longitudinal tension, transverse tension, longitudinal compression, transverse compression, and in-plane shear. The

(a). With tested constitute model (Bi-modulus)



(b). Without Bi-modulus ($E_c = E_t$)

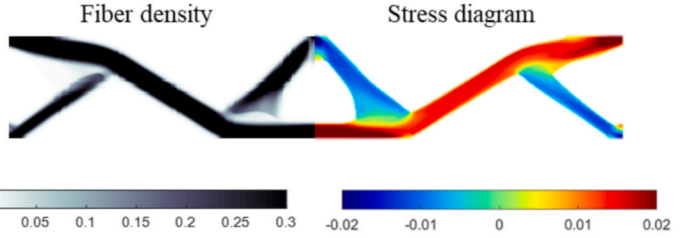
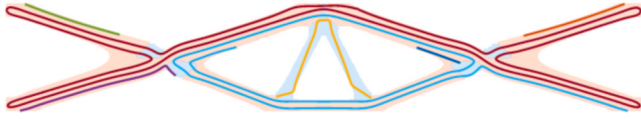


Fig. 21. Re-optimized designs based on the experimentally identified constitutive model: (a) with bi-modulus behavior; (b) without bi-modulus behavior ($E_c = E_t$).

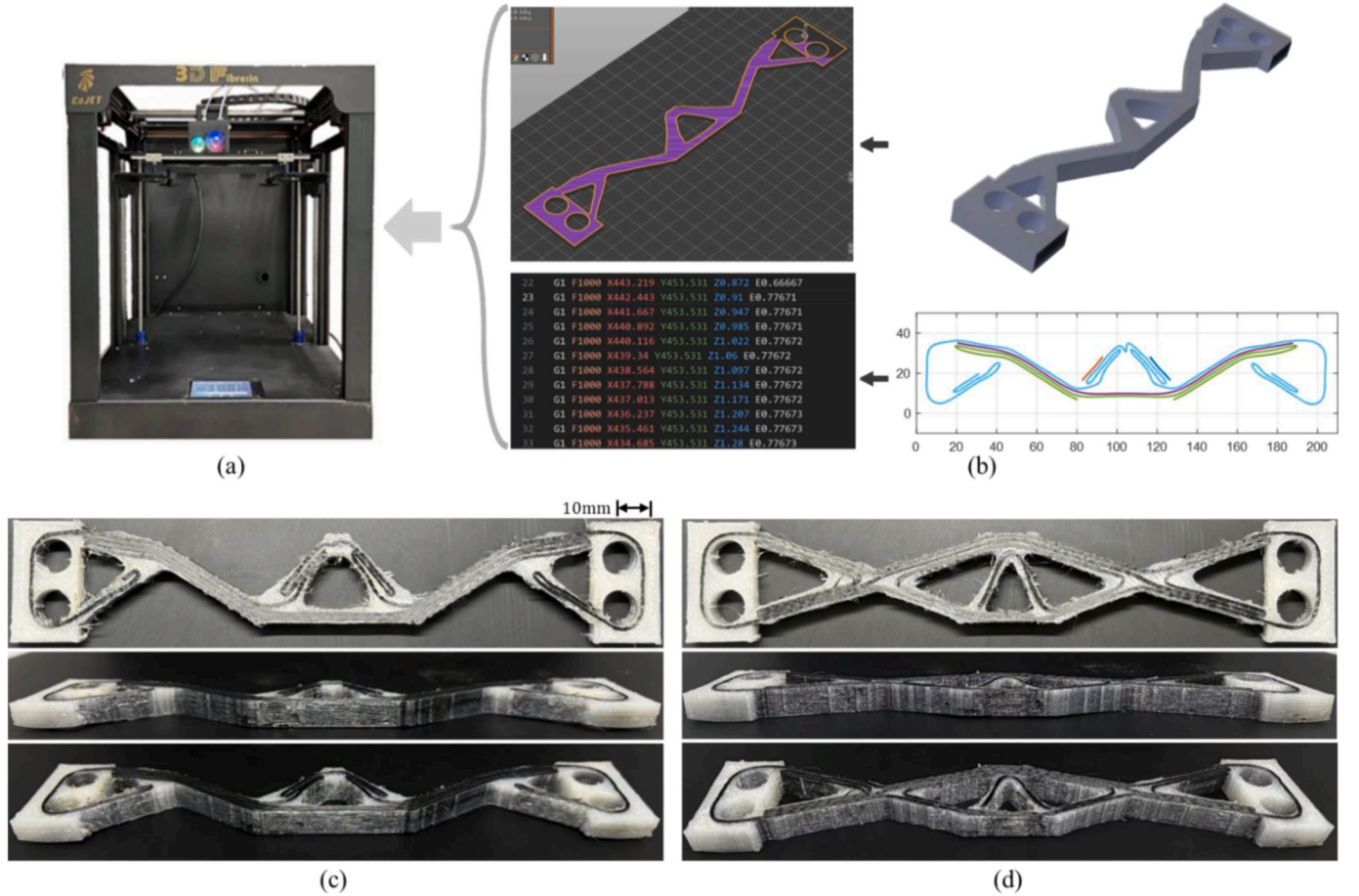


Fig. 22. Experimental fabrication workflow and printed specimens for Cases. (a) Printing system; (b) STL based slicing and fiber toolpath generation via iso-contour extraction and G-code conversion; Fabricated specimens (c) with bi-modulus consideration, and (d) without bi-modulus consideration, shown from multiple perspectives.

Table 5
Printing parameters.

Parameter	Value
Matrix nozzle temperature (°C)	275
Fiber nozzle temperature (°C)	275
Platform temperature (°C)	100
Matrix printing speed (mm/s)	50
Fiber printing speed (mm/s)	20
Layer thickness (mm)	0.3
Extrusion flow	100%

dimensions of the corresponding specimens and the associated testing standards are summarized in Table 2. The printed specimens and the tested specimens after mechanical characterization are shown in Fig. 20.

The experimentally measured effective moduli are summarized in Table 3. It is evident that the 3D-printed CFRCs exhibit clear tension-compression asymmetry and directional anisotropy, as reflected in the distinct tensile and compressive moduli as well as the differences between the longitudinal and transverse moduli, consistent with the results reported by Hou et al. [55].

By substituting the experimentally measured moduli in Table 3 into Eq. (11), the unknown constituent parameters in the interpolated model can be identified. The resulting parameter values are listed in Table 4. Due to the difficulty of obtaining stable transverse strain during the experiments, Poisson's ratio was not measured directly. Instead, it was estimated using the rule of mixtures, following the same treatment as

Hou et al. [55].

8.2. Optimization with experimentally identified parameters

After obtaining the experimentally identified constitutive parameters, the double clamped beam case was re-optimized based on the corresponding constitutive model. The optimization results are shown in Fig. 21.

As shown in Fig. 21(a), the optimized structure obtained using the experimentally identified bi-modulus constitutive model exhibits a structural layout similar to that of numerical case DD.3.3. This can be mainly attributed to the fact that the experimentally identified ratio of compressive to tensile modulus along the fiber direction is 0.136, which is closer to the tension-compression modulus ratio prescribed in DD.3.3. Consequently, the two cases exhibit consistent optimization trends. For comparison, a reference design neglecting tension-compression asymmetry was also generated by retaining the experimentally measured tensile modulus and setting the compressive modulus equal to the tensile modulus, similar to case DD.3.1.

After optimization, the printing preparation workflow is illustrated in Fig. 22. Specifically, the geometric boundaries of the topology optimized results are exported in stereolithography (STL) format and imported into the slicing software. In parallel, the optimized fiber trajectories are generated by extracting the iso-contours of the optimized geodesic fields and subsequently converted into executable G-code for toolpath implementation. Both specimens share identical

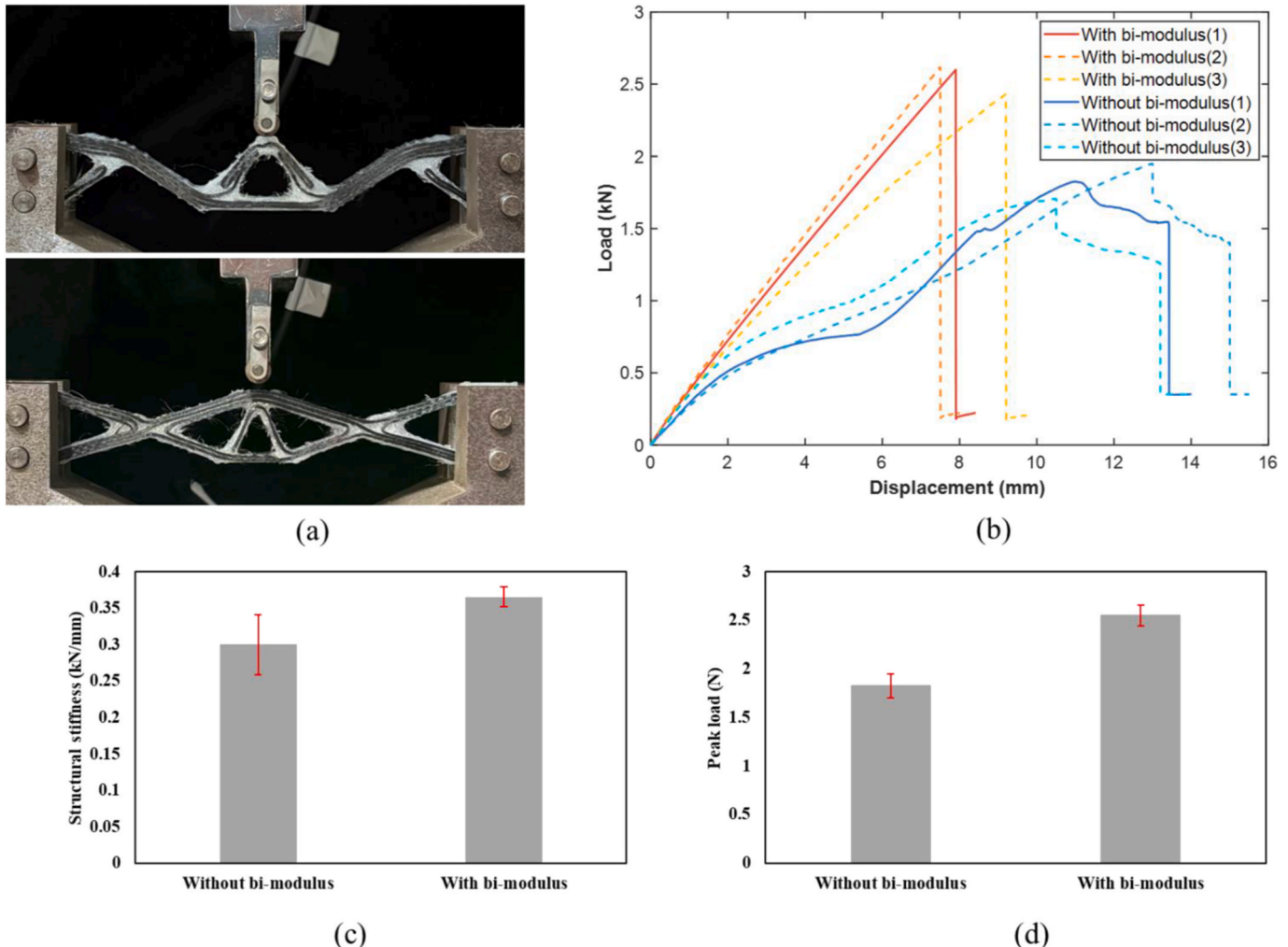


Fig. 23. Experimental process and results: (a) experimental setup; (b) load-displacement responses; comparison of (c) Structural stiffness and (d) Peak load.

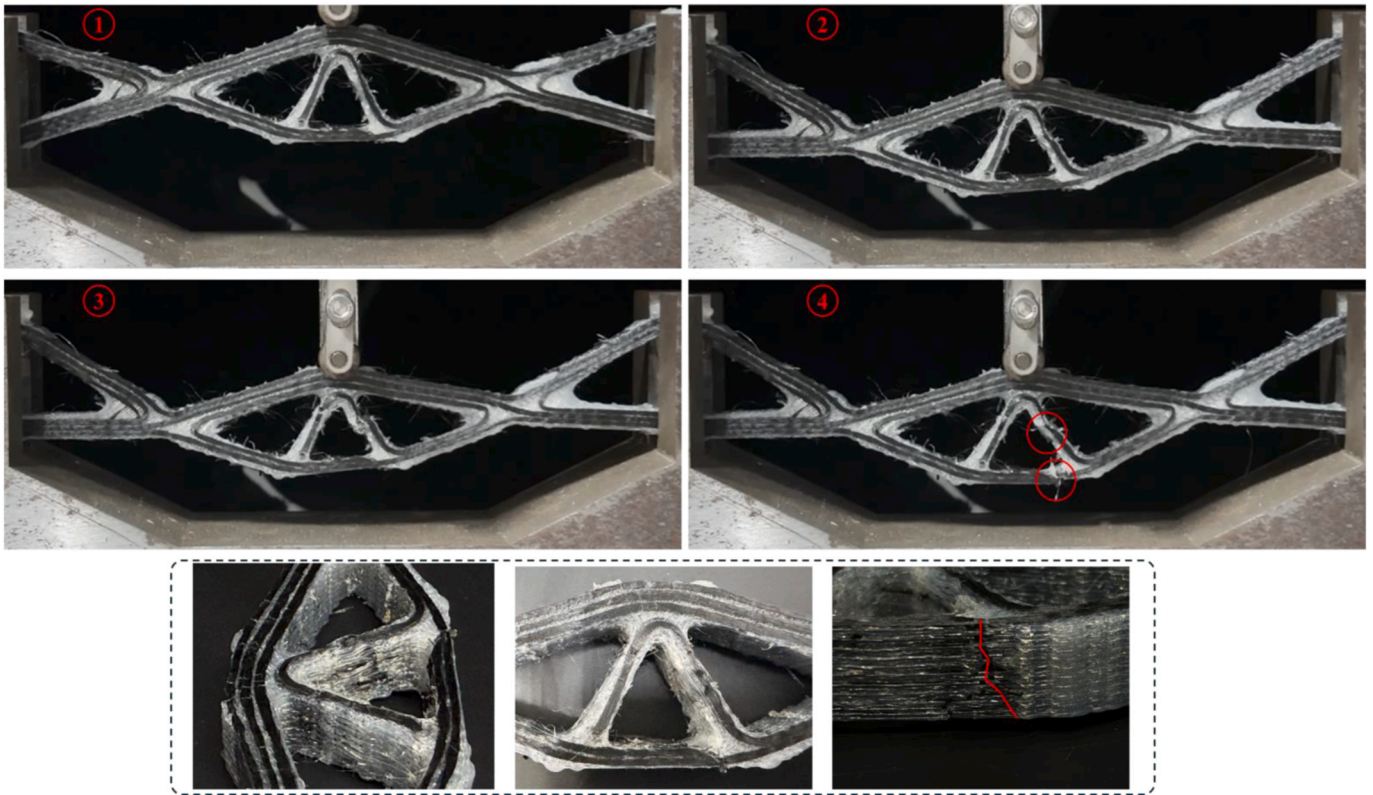


Fig. 24. Deformation and failure evolution of the printed specimen for the design without bi-modulus consideration.

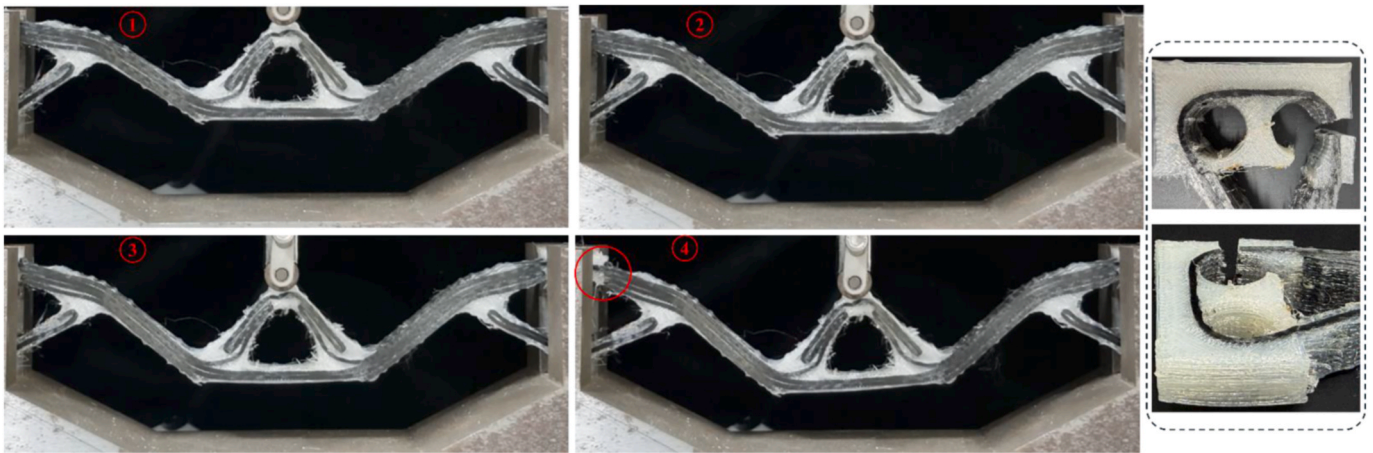


Fig. 25. Deformation and failure evolution of the printed specimen for the design with bi-modulus consideration.

geometric constraints, with overall dimensions of $210 \text{ mm} \times 39 \text{ mm} \times 10 \text{ mm}$. The printing parameters used for fabrication are summarized in Table 5.

8.2.1. Experimental testing

Fig. 23 presents the compression responses of the two optimized designs, with three repeated tests performed for each case. The specimens exhibit comparable average masses, with $28.36 \pm 0.22 \text{ g}$ for the design with bi-modulus consideration and $29.12 \pm 0.26 \text{ g}$ for the design without bi-modulus consideration. During the test, a downward displacement is applied at the midspan of the top surface at a constant loading rate of 2 mm/min , while both ends are clamped, as illustrated in Fig. 23(a), consistent with the boundary conditions used in the optimization. The corresponding load-displacement curves are shown in

Fig. 23(b). In the small-deformation regime, both designs exhibit an approximately linear increase in load with displacement. However, the design without bi-modulus consideration shows noticeable fluctuations before reaching the peak load, indicating unstable load transfer accompanied by progressive damage evolution and local buckling. By contrast, the design with bi-modulus consideration maintains a consistently higher slope and achieves a significantly greater load-carrying capacity.

Quantitative comparisons are presented in Fig. 23(c) and (d). Within the displacement range of $0.3 - 1 \text{ mm}$, the optimized structure without bi-modulus consideration exhibits a stiffness of $0.301 \pm 0.041 \text{ kN/mm}$, while that with bi-modulus consideration reaches $0.365 \pm 0.015 \text{ kN/mm}$, corresponding to an increase of approximately 21.3%. Moreover, the average peak load increases from $1.828 \pm$

0.122 kN to 2.550 ± 0.104 kN, representing an improvement of approximately 39.5%.

8.2.2. Failure mode analysis

From the experimental load–displacement results, the design without bi-modulus consideration (Fig. 24) exhibits a multi-stage progressive failure behavior dominated by compression-induced instability. Following an initially stable response (①), local buckling initiates in the compressive region, leading to stiffness degradation while the global structure remains stable (②). Further loading causes damage accumulation in the buckled region, triggering the first instability and a sudden drop in load-carrying capacity (③). Owing to load redistribution to tensile-dominated regions, a residual load-bearing capacity is maintained until final rupture of the slender bottom tensile strut (④).

In contrast, the design with bi-modulus consideration (Fig. 25) effectively suppresses premature compression-driven buckling, which explicitly accounts for the tension–compression anisotropy. By aligning continuous fibers predominantly along tensile load paths and enhancing the stability of compressive regions, the onset of local instability is significantly delayed, and fracture was finally observed only at the interface between the specimen and the fixture (④). As a result, this design achieves a substantially higher peak load and a more stable load–displacement response prior to failure (①–③), highlighting the critical role of tension–compression anisotropy in the structural design of continuous-fiber-reinforced systems.

More fundamentally, this study demonstrates that explicitly embedding the tension–compression asymmetry of CFRCs into the structural design process enables a mechanism-aware redistribution of load paths, thereby preventing instability-dominated failure and leading to more robust and efficient CFRCs.

9. Conclusion

This paper proposes a dual-field framework for the co-optimization of structural topology and continuous fiber paths and provided a post-processing pipeline that converts the optimized fields into fabrication-ready toolpaths. The main contributions are:

- Elimination of the mismatch between the optimized fiber orientation and the generated fiber path orientation. By optimizing differentiable geodesic fields, the proposed framework directly embeds the orientation of the final printable fiber paths into the optimization process, thereby eliminating the mismatch commonly introduced by conventional post-processing procedures for fiber path generation.
- Ambiguity-robust fiber path. Two differentiable geodesic fields with controllable gradients were introduced to construct vector fields whose equivalent streamlines are globally consistent, resolving the issue of π -periodic orientation ambiguity and improving overall path continuity.
- Content-aware spacing. A fiber-content variable coupled with the geodesic field gradients explicitly encoded spacing among the printable paths, enabling control of local fiber volume fraction under a global cap (via a p -norm approximation).
- Bi-modulus constitutive modeling. The WCM formulation incorporated tension–compression asymmetry during optimization, yielding designs that better reflect anisotropic composite behavior.
- Manufacturability constraints. Minimum length scale constraints and a maximum fiber-content constraint were enforced throughout, leading to layouts that are both performant and printable.
- Printable toolpaths. A practical post-processing workflow (contour extraction $\rightarrow$ stitching/cleanup $\rightarrow$ endpoint pairing $\rightarrow$ B-spline smoothing) produced continuous, non-interfering paths suitable for material extrusion 3D printing.
- Experimental validation. Prototype fabrication and mechanical testing calibrate the bi-modulus parameters and validate the predicted performance gains of the optimized designs.

Despite these advances, several avenues remain:

- Modeling scope. The study is limited to small-strain elasticity in 2D; extension to 3D, large-deformation regimes, multi-load cases, and failure/strength constraints are needed.
- Path kinematics. Curvature/acceleration limits and print-order optimization were only approximated via smoothing and greedy pairing; a full trajectory-planning stage (with collision and speed constraints) would improve printability.

Overall, the proposed dual-field co-optimization framework provides a practical route from design fields to printable fiber paths, narrowing the gap between numerical design and real manufacturing.

CRediT authorship contribution statement

Yifan Guo: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation. **Chang Su:** Validation, Investigation. **Jikai Liu:** Writing – review & editing, Supervision, Methodology, Investigation, Funding acquisition, Conceptualization. **Shuzhi Xu:** Writing – review & editing, Validation, Methodology, Investigation. **Takayuki Yamada:** Writing – review & editing, Funding acquisition.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Data availability

Data will be made available on request.

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