

Self-support structure topology optimization for multi-axis additive manufacturing incorporated with curved layer slicing

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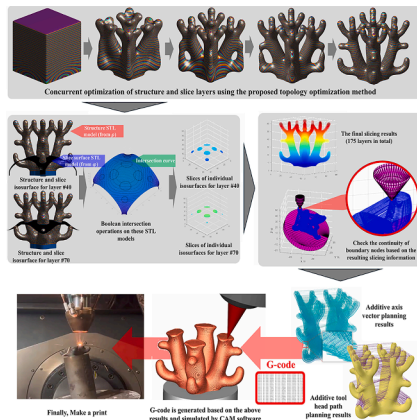
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GRAPHICAL ABSTRACT



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ABSTRACT

Multi-axis additive manufacturing significantly surpasses traditional 3-axis systems by utilizing multiple axes of motions that constructs complex three-dimensional structures with reduced need of supports. However, process planning for the curved layer slicing determines the interactions between the part and supports, and consequently, self-support topology optimization requires a numerically tractable process planning algorithm to derive the sensitivities, which however, has

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yet been achieved. To fill the gap, we develop a structural topology optimization method for multi-axis additive manufacturing, which features in achieving the self-support effect by deeply incorporating the curved layer slicing. Specifically, a process scalar field is generated on top of a domain of pseudo-densities by solving a heat diffusion equation and a Poisson equation, through which the geodesics included in the scalar field facilitate the curved layer slicing and any geometric information about the layers are derivable on the pseudo-densities because of the tractable numerical processing routine. Then, self-support constraints for multi-axis additive manufacturing can be established by measuring the curved layer normals and the part boundary gradients. Coupled with the density variables for topology optimization, our proposed method could concurrently optimize the part structure and its curved slicing pattern, maximizing the structural physical performance while eliminating the need of supports. Finally, we validated and discussed the effectiveness of our method through a series of numerical tests and provided a workflow to show the strong correlation between our optimized results and the actual spatial paths.

1. Introduction

Topology optimization, as an advanced design method, aims at redistributing materials to minimize structural weight while meeting mechanical requirements [1,2]. With over 40 years of development [3–7], people could now apply topology optimization to solve a wide range of problems [8,9,13,14,15,16] or in various scenarios [10,11,12,17]. Structures generated through topology optimization often exhibit complex and organic geometries. Owing to these intricate geometric features, the optimized structures can maintain decent performance while being lighter. However, traditional manufacturing methods frequently encounter challenges when handling such complex structures. The emergence of Additive Manufacturing (AM) [18–21], also known as 3D printing, has enabled the widespread application of topology optimization for fixing the manufacturability issues. With AM, objects are built layer by layer through adding materials. Compared to traditional subtractive manufacturing, AM provides the ever increased design flexibility, enabling the production of complex geometries, minimizing material waste, and consolidating functional components in a single manufacturing process.

Undoubtedly, AM and topology optimization are working closely [22,23]. Even so, AM processes still raise manufacturability restrictions to design, leading to the necessity of developing topology optimization algorithms dedicated to AM. For instance, ensuring geometric features not overly complex is essential to avoid processing failures caused by insufficient resolution of the printer nozzle [24]. Adjusting part geometry to mitigate residual stress [25–27] and distortion [28,29,30,31,32,34] for laser-based AM processes facilitates eliminating cracks and over-deformation. Furthermore, incorporating the anisotropy material feature [36,37] and integrating it with printing paths for structural optimization [33,35,38,39,40,41,42,43,44] can ensure the optimized structural performance approaching the actually tested performance.

In both academia and industry, most AM processes still rely on 3-axis machines, wherein materials are deposited along a fixed Z-axis. For parts with complex geometries, additional support structures or multiple manufacturing operations are often necessary, inducing increased manufacturing difficulties and costs. Hence, for topology optimization design, it is crucial to consider self-supporting features on structures to mitigate surface quality issues and reduce additional machining costs for support structure



Fig. 1. The DMG LASERTEC 65 3D with a multi-axis additive manufacturing system.

removal [45,46,47,48,49]. Generally, two methods are employed to achieve self-supporting design: the projection-based method and the boundary gradient-based method. The projection-based method is widely employed in density-based topology optimization. This method involves designing specific filter kernels to make the filtered density field with self-supporting characteristics [46,47,50,51,52,53]. The boundary gradient-based method is employed in both density-based topology optimization methods and level-set method with clear boundary expressions [54,55,56,57], identifying local boundary angles to ensure they are less than the prescribed overhang angle threshold. Both methods can be employed in the form of self-supporting geometric constraint, incorporated into topology optimization algorithms implicitly or explicitly. However, such self-supporting constraints may significantly limit design freedom, and thus compromising structural performance [58]. It is worth noting that a method exists to balance these two aspects: controlling only the self-support of enclosed voids while allowing support structures to appear in other parts of the design [59,60]. This approach effectively reduces the need for self-supporting regions, thereby allowing for greater design freedom. Additionally, it minimizes the use of support structures, ensuring that any necessary supports are located in easily accessible external areas, which reduces post-processing complexity and printing costs.

In recent years, multi-axis AM is gaining attention (one example machine is shown in Fig. 1). As an advanced AM technology, it utilizes multiple axes of motions to control the material deposition process, enabling the construction of complex three-dimensional structures. Compared to traditional 3-axis AM, multi-axis AM enhances the manufacturing flexibility, allowing the fabrication of more complex geometries. Moreover, it enables better control over material deposition paths, thereby enhancing manufacturing quality and efficiency. The applications of multi-axis AM spans among aerospace, automotive, biomedical, and other fields [61].

With the fast development of multi-axis additive manufacturing, the associated design methods are emerging. From the topology optimization community, there are several research efforts focused on multi-axis AM. For instance, concurrent printing direction and structural geometry optimization has been explored to achieve the optimal performance of a self-supporting structure [62,63,64]. However, such works are still restricted to planar-layer based printing with 3-axis machines, failing to exploit the degrees of freedom offered by the rotating axes. The work [65] introduces the idea of printing along the principal stress directions with six degrees of freedom, evoking thoughts of concurrent optimization of fiber paths and structural topology. However, the generated paths lack crucial process information, leading to numerous challenges of post-processing, and the actual feasibility in printing was not validated. The work [66] attempted to optimize process regions alongside structural optimization to achieve design for multi-axis AM. However, this effort merely provides process regions and their sequences, while the crucial slicing and path information, as well as the collision evaluation, are all missing. Additionally, the so-called ‘time optimization’ in this work results in isolated islands of the generated processing regions. Despite a ‘continuity constraint’ was introduced to address this issue, the validity of these constraints remains questionable in general sense. The other study [67] encounters similar issues when dealing with the residual deformation control for multi-axis Directed Energy Deposition (DED) processing. The inherent strain method for predicting residual deformation is especially sensitive to the slicing and path condition. The study [68] extends the work of [63] by optimizing the printing angles within multiple regions to achieve self-supporting designs for multi-axis AM. However, the pattern of dividing the design domain is pre-defined and invariable alongside the optimization, and such type of regional optimization severely restricts the space of design freedom. Besides, the sequence information is missing about the regions, causing unevaluable collision-free condition of the printing laser head. It is also acknowledged that involving tens of discrete regions is challenging for the current multi-axis 3D printing technique, given the non-trivialness of ensuring manufacturing quality at the interface areas.

We can see from the above survey that, topology optimization for multi-axis AM currently lacks an explicit interaction with the process condition, e.g., the slicing and path information, causing difficulties in accurately evaluating the manufacturability and guiding the 3D printing validation. In fact, for existing DED-based AM, regardless of three-axis or five-axis manufacturing, the material deposition process strictly follows the layer-by-layer pattern. Regarding the layer-by-layer deposition, several processing requirements must be carefully considered: (i) The layer thickness should be as uniform as possible (see Fig. 2a). (ii) The deposition process should be as continuous as possible (see Fig. 2b). (iii) Additionally, it is crucial to avoid excessively curved layers that might induce printing failure or printing nozzle collision (see Fig. 2c). Neglecting these factors while solely relying on geometric and temporal information

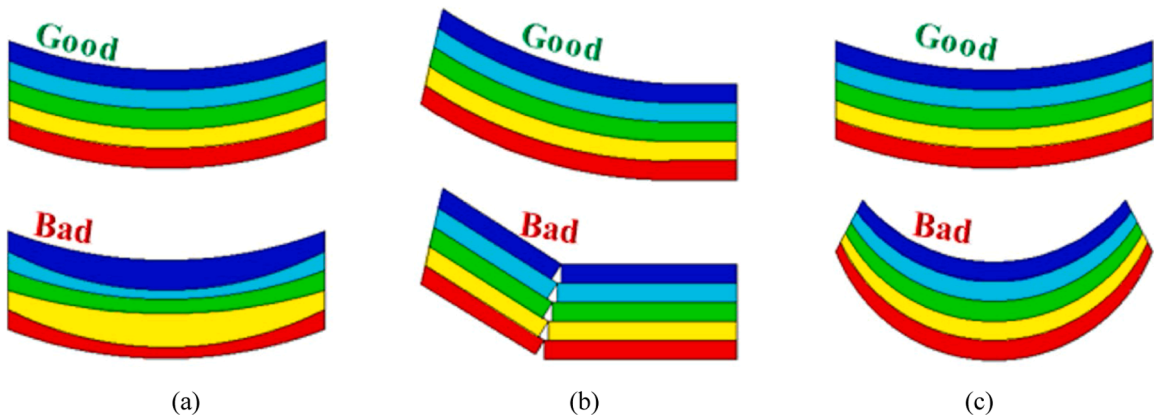


Fig. 2. Four typical slicing issues: (a) Non-uniform layer thickness; (b) Discontinuous slicing; (c) Overly curved layers.

for process planning is over simplistic.

Apparently, optimizing slicing condition based on the aforementioned requirements is not a straightforward task. Research has been conducted on slicing optimization for multi-axis AM. The aim of these works is often to provide slices and paths for printing complex geometries while addressing certain AM constraints (such as self-supporting, collision avoidance, material anisotropy, etc.) [69,70,71,72]. Currently, for most given geometric configurations, existing slicing optimization algorithms can already provide high-quality results in terms of slicing geometry. However, in certain special cases, due to the organic topological features of the slicing object, it may be impossible to identify a suitable slicing solution that satisfies all the corresponding AM constraints. To illustrate this point, we present a sample 2D case in Fig. 3. In Fig. 3, the inner top of the circular ring exhibits a significant overhang feature (a common geometric feature appearing in topology optimization results). If forced to print, a low-quality overhang surface would be produced with FDM process, while for DED-AM, this area would experience severe powder leaking and fail to shape the overhang surface [81]. Without structure modification, curved layer generation would very likely cause collisions between the laser head and the already built part.

From the perspective of the authors, achieving a balance between structural performance and manufacturing feasibility is a good strategy to tackle the challenges mentioned above. In other words, we aim at achieving feasible process plans for multi-axis AM by concurrently optimizing the structures while minimizing sacrifices to structural performance due to manufacturability considerations (Fig. 3). Driven by this idea and inspired by numerous excellent works from computer graphics, we introduce the topology optimization algorithm incorporated with continuous and uniform 3D printing layer generation. The process scalar field and the structural topological density field will be concurrently optimized and deeply interacted by the manufacturability constraints. The optimization objective is to maintain structural performance while minimizing weight, and the optimization constraints involve generating continuous and uniform layers while ensuring collision-free and support-free printing.

Finally, the proposed work offers several application advantages. Firstly, the algorithm builds upon traditional topology optimization frameworks by solving two additional well investigated linear physical problems and performing a series of linear operations (such as convolutions) without significantly increasing computational complexity. Additionally, the optimization algorithms used in our approach can adopt the state-of-the-art gradient-based large-scale non-convex optimization solvers (such as MMA), and thus ensuring good convergence of the optimization. We propose a concise and robust optimization framework that guarantees the scalability of the algorithm.

2. The curved layer slicing algorithm

In this study, our manufacturing environment is a 5-axis DED-type AM machine with a rotating platform. Unlike traditional 3-axis AM machines that only allow the nozzle motions along the XYZ axes, this type of machine features an additional U axis and V axis of rotating the platform, as illustrated in Fig. 4. By rotating the platform, we can mitigate the influence of gravity and achieve the printing of large overhang structures. Then, slicing the part to create the planar/curved layers would be crucial.

In this work, a scalar field with uniform gradient is employed to represent the sliced layers; see Fig. 5. Specifically, given an arbitrary scalar field φ , Eq. (1) is satisfied at the majority of the domain, so that the absolute value at any point indicates its shortest

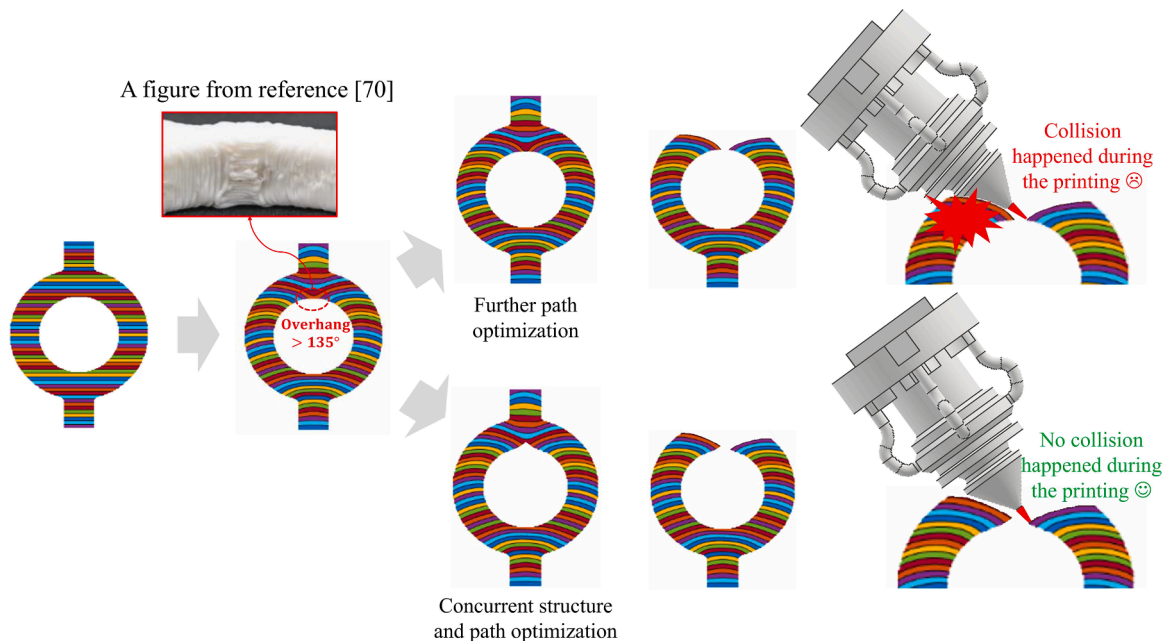


Fig. 3. The illustration of a difficult-to-process part only by process planning optimization.

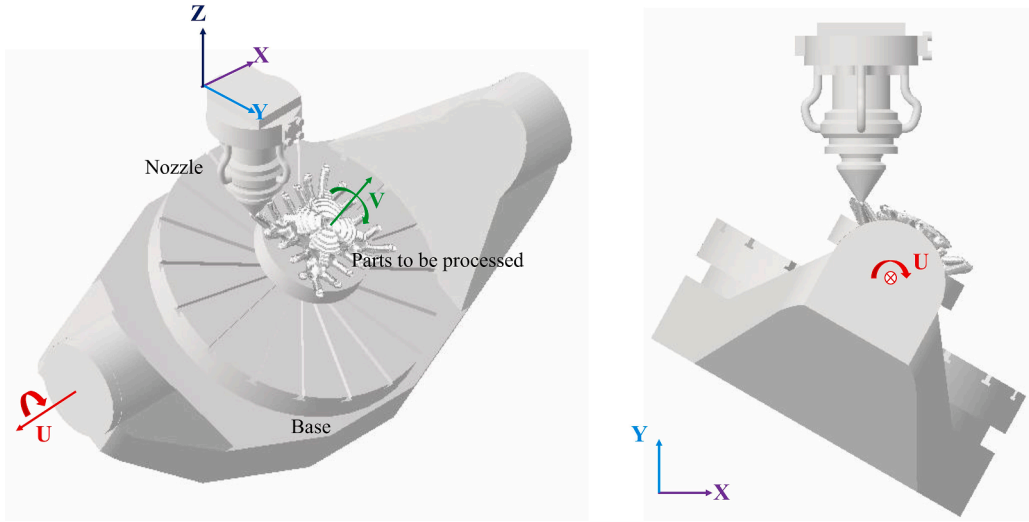


Fig. 4. The 5-axis AM system considered in this work. The left side represents a diagram of degrees of freedom in 3D scenario, while the right side illustrates the same in 2D scenario.

distance to the zero-level contour.

$$\|\nabla\varphi(\mathbf{X})\| = 1 \quad (1)$$

By assuming the printing bed as zero-level profile, the layers can be identified by extracting the scalar field profiles with constant intervals, i.e. the profiles with scalar values of 0, d , $2d$, $3d$, ..., nd , where d represents the constant interval between any two adjacent layers.

In the following sub-Sections, we will present the heat method [78] for solving the shortest path problem on design domain Ω_p . A crucial insight lies in the two-stage distance computation: first, identify the directions that distances are increasing, and then, build the geodesic field by regulating the distance gradients. The aforementioned aspects will be elaborated on in details.

2.1. Temperature field with fabrication sequence information

For a medium with certain thermal conductivity, thermal energy diffuses from regions of higher temperature to those of lower temperature (the phenomenon known as thermal diffusion). By observing temperature variations across the design domain, we can obtain sequence information of temperature propagation. In this work, we utilize the following linear diffusion equation with Dirichlet boundary condition to obtain the distributed temperature field:

$$\begin{cases} \nabla \cdot (\kappa \nabla T) = 0 \\ T = T_{\text{start}} \text{ on } \Gamma_{\text{start}} \\ T = T_{\text{end}} \text{ on } \Gamma_{\text{end}} \\ \nabla T \cdot \mathbf{n} = 0 \text{ on } \Gamma \setminus (\Gamma_{\text{end}} \cup \Gamma_{\text{start}}) \end{cases} \quad (2)$$

where ∇ is the differential operator, and $\mathbf{n}$ is the outward normal vector of the boundary, Γ indicates for the boundary of the domain.

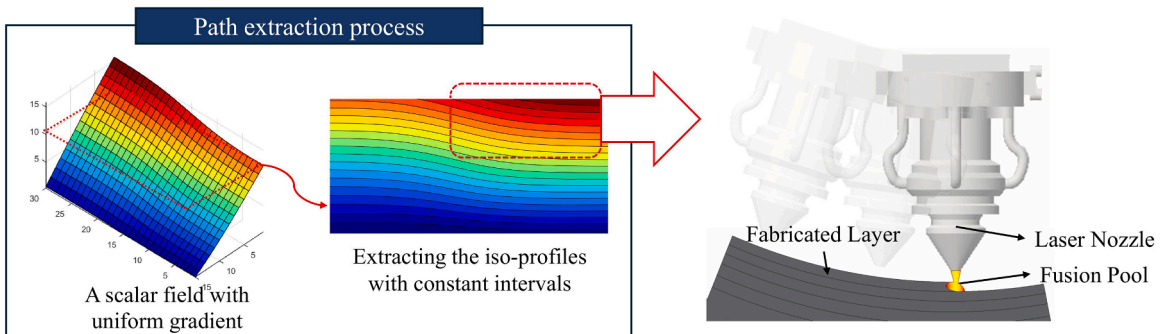


Fig. 5. The basic idea for curved layer generation.

This Partial Differential Equation (PDE) represents steady-state heat conduction within a heterogeneous medium, featuring with varying thermal diffusivity κ . Γ_{start} indicates the boundary where the printing starts, and Γ_{end} indicates the boundary where the printing ends.

Then, let us consider an example with 2D rectangular domain. We designate the entire bottom as the printing platform (Γ_{start}) and assign it with a higher temperature (T_{start}). Subsequently, we set a lower temperature (T_{end}) at the top of the design domain (Γ_{end}) to represent the boundary that ends the printing. Finally, the materials with homogeneous/heterogeneous κ are assigned for the design domain. Consequently, heat will diffuse from bottom to top, while the exact diffusion paths are dependent on the property distribution of the medium (see Fig. 6). Nevertheless, the temperature field exhibits continuous changes with spatial variations.

Typically, for a given manufacturing scenario, the initial Dirichlet boundaries (Γ_{start} and Γ_{end}) can be fairly determined according to the design domain arrangement. Hence, when solving Eq. (2), a fixed boundary condition can be assumed. The variation in temperature field is only affected by the changeable thermal diffusivity κ of the medium. Here, we construct a pseudo-density field τ and utilize it for thermal diffusivity interpolation to achieve the tunable properties. For instance, for the e^{th} element within domain Ω_D , we have:

$$\kappa_e = \kappa_{\min} + \tau_e(\kappa_0 - \kappa_{\min}) \quad (3)$$

Eq. (3) is very similar to the classic SIMP interpolation, where $\kappa_{\min}$ represents the thermal diffusivity for void elements, and κ_0 represents the thermal diffusivity for solid elements. However, unlike SIMP, Eq. (3) does not penalize the pseudo elemental density, because a ‘clear’ solid/void distribution is not required for heat conduction analysis. Any intermediate density value may exist without imposing a clear physical meaning, for which more details will be elaborated in the subsequent sections. Then, Eq. (2) is reformulated as a result of solving the aforementioned PDE using FEM as:

$$\mathbf{K}_{\text{th}} \mathbf{T} = 0 \quad (4)$$

where $\mathbf{K}_{\text{th}}$ represents the thermal stiffness matrix, and $\mathbf{T}$ is the global temperature vector.

Last but not least, the Dirichlet boundary setting is flexible and can be applied to any location of any boundary. Additionally, while Γ_{start} must be defined, it is sometimes possible to omit Γ_{end} , allowing for a more flexible path distribution.

2.2. Pseudo-scalar field with uniform gradient information

In the last subsection, by solving the heat diffusion problem with specific Dirichlet boundary conditions, a temperature field could be obtained, but the temperature gradient distribution is non-uniform. Recalling that for 5-axis printing, materials are added layer by layer with close-to-uniform layer thickness to ensure the satisfactory quality. Therefore, instead of directly using the temperature field, it is needed to build a replacement scalar field, also denoted as φ , which possesses similar properties to the temperature field (approximately the same gradient directions and Dirichlet boundary values) and very importantly, the uniform gradient norms (similar to a geodesic distance field). Then, the corresponding spatial and temporal information for manufacturing can be obtained from this scalar field φ . To achieve a satisfactory φ , we begin by normalizing the temperature gradients to achieve a unit temperature gradient ω :

$$\omega = \frac{\nabla T}{\|\nabla T\|}, \quad (5)$$

and then, forcing the scalar field φ to approach the following condition:

$$\nabla \varphi = \omega, \quad \varphi|_{\gamma} = T_{\text{start}} \quad (6)$$

where γ indicates some subset of Ω_D , here, indicating the printing platform (Γ_{start}). The main purpose of making $\varphi|_{\gamma} = T_{\text{start}}$ is to ensure the initial printing direction consistency between φ and T , while enforcing $\nabla \varphi = \omega$ is to guarantee the printing orientation consistency between φ and T . Typically, the scalar field φ can be obtained by solving the following optimization model:

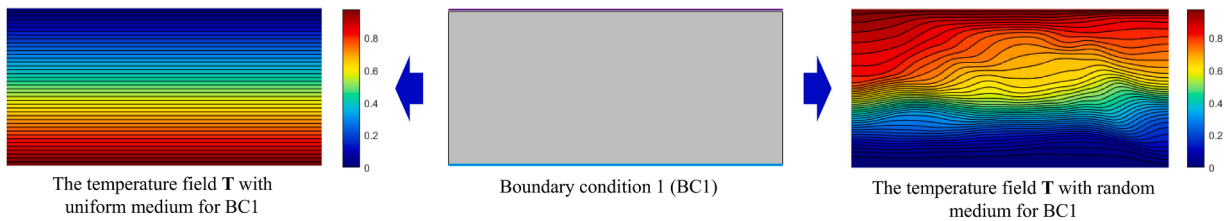


Fig. 6. The temperature distribution for the medium with uniform heat conductivity properties (left) and random heat conductivity properties (right) under boundary condition 1. The color bar means the normalized temperature.

$$\begin{cases} \text{find : } \varphi \\ \text{minimize : } F = \int (\nabla \varphi - \omega)^2 d\Omega_D \\ \varphi = T_{\text{start}} \text{ on } \Gamma_{\text{start}} \end{cases} \quad (7)$$

Alternatively, we can solve the problem in Eq. (7) using variational theory [78]. By substituting the objective function F from Eq. (7) into the Euler-Lagrange equation, we arrive at:

$$\frac{\partial F}{\partial \varphi} = \nabla \cdot \frac{\partial F}{\partial \nabla \varphi} = \nabla \cdot \frac{\partial (\nabla \varphi - \omega)^2}{\partial \nabla \varphi} \quad (8)$$

Since in Eq. (8), F is only a function of $\nabla \varphi$, we have $\frac{\partial F}{\partial \varphi} = 0$. Therefore, Eq. (8) can be easily expressed as:

$$0 = \nabla \cdot (\nabla \varphi - \omega) \quad (9)$$

Thus, solving the optimization problem in Eq. (8) can be transformed into solving the following equation:

$$\begin{cases} \Delta \varphi = \nabla \cdot \omega \\ \varphi = T_{\text{start}} \text{ on } \Gamma_{\text{start}} \end{cases} \quad (10)$$

So, we could directly approximate the φ by solving the following Poisson equation by FEM:

$$\mathbf{K}_{\text{po}} \varphi = \mathbf{Q}_{\text{po}}(T) \quad (11)$$

where $\mathbf{K}_{\text{po}}$ represents the global Laplace matrix, φ is a vector, and the load $\mathbf{Q}_{\text{po}}$ is a temperature dependent vector.

Fig. 7 illustrates the process of transforming the temperature vector $\mathbf{T}$ into the φ . For better conceptual clarity, in the subsequent sections of this paper, we refer to this vector φ as the process field. In this process field, it not only contains temporal information (the sequence of fabrication indicated by the scale value) but also spatial information (the geodesic distance from the printing platform).

In conclusion, the heat method is robust, efficient, and simple to implement, since it involves solving a pair of standard sparse linear systems. These systems can be factored once and subsequently solved in near-linear time, substantially reducing amortized cost. Moreover, the heat method is differentiable. By solving two adjoint equations, we can efficiently obtain sensitivity information on the pseudo-densities τ . Finally, the process field, indicating geodesic distances carries the continuous processing time information.

3. Self-support design for the structure with curved layer slicing

3.1. The design workflow

The proposed realization is illustrated in Fig. 8. To begin with, for the structure design, the typical density filter with radius R_1 will be used for the density design field μ to avoid the checkboard phenomenon to obtain the filtered field $\tilde{\mu}$. Then, $\tilde{\mu}$ could be expressed as:

$$\tilde{\mu} = M(\mu, R_1) \quad (12)$$

where the $M(\cdot)$ is the smoothing function, and R_1 is the smoothing radius. It could be any smoothing filter commonly found in the field of topology optimization (like the PDE filter or convolution filter and so forth), for which the detailed explanations could refer to [75,76]. Then, the approximated Heaviside projection [74] is applied to $\tilde{\mu}$ to obtain the 0–1 field ρ :

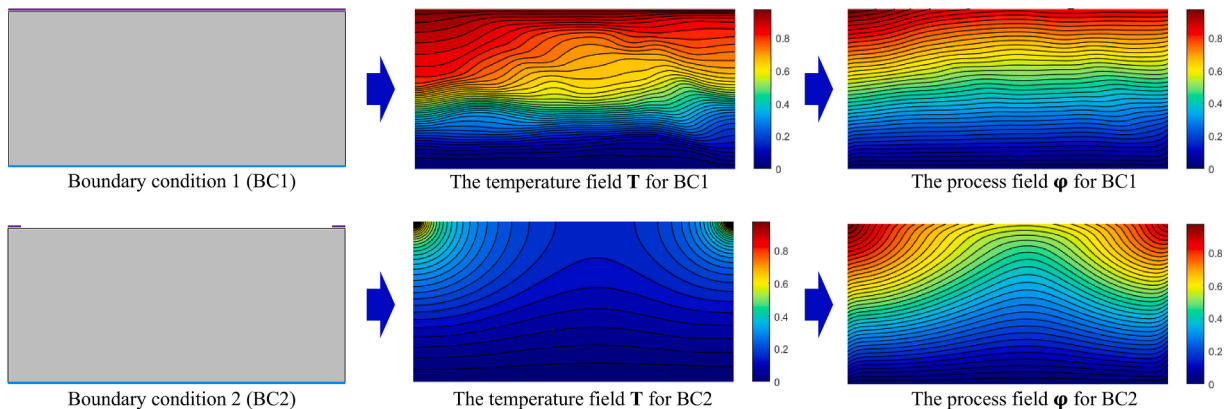


Fig. 7. The derivation of a scalar field with uniform gradient norms subject to different boundary conditions.

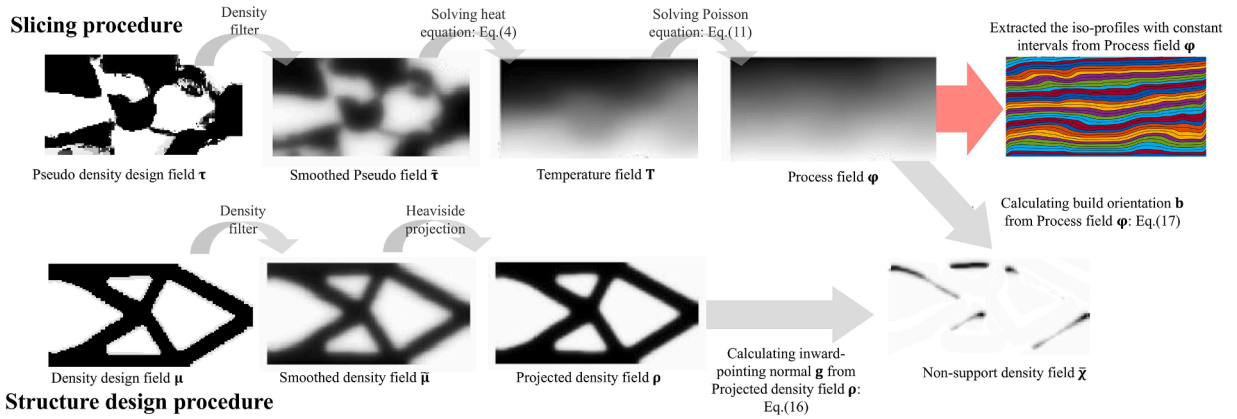


Fig. 8. The flowchart of non-support area analysis.

$$\rho_e = \frac{\tanh(\beta\delta) + \tanh(\beta(\tilde{\mu}_e - \delta))}{\tanh(\beta\delta) + \tanh(\beta(1 - \delta))} \quad (13)$$

where β is the parameter to control the projection sharpness, and δ is the threshold value. In this work, the continuum strategy [77] of modifying the sharpness factor will be adopted, and the threshold value is set as 0.5.

Regarding the process planning procedure, it is worth noting that by solving the Poisson equation in Eq. (9), we can only obtain an approximation of the geodesics. The accuracy of this approximation largely depends on the distribution of the input temperature field. Generally, less fluctuating input temperature gradients lead to a better accuracy. Under given boundary conditions, the spatial fluctuation of temperature gradients can be suppressed by preventing the thermal diffusivity from varying fiercely in space. Thus, a smoothing filter is also applied to the pseudo-density field τ :

$$\tilde{\tau} = M(\tau, R_2) \quad (14)$$

And for the e^{th} element, the Eq. (3) will be further modified as:

$$\tilde{\kappa}_e = \kappa_{\min} + \tilde{\tau}_e(\kappa_0 - \kappa_{\min}) \quad (15)$$

Then, the modified field $\tilde{\kappa}$ will be used for the heat diffusion analysis with given boundary conditions through Eq. (4) and Eq. (11) to obtain the temperature vector $\mathbf{T}$ and process vector φ .

3.2. The self-support geometric constraint

Identifying the local boundary angle is an effective way to design self-support structures. In this paper, this idea is inherited to identify the overhang surfaces and impose the control. The overhang angle θ is defined as the angle between the build orientation $\mathbf{b}$ and inward-pointing normal $\mathbf{g}$ at a boundary point. The inward-pointing normal is evaluated by the gradient of the projected density field ρ :

$$\mathbf{g} = \left(\frac{\partial \rho}{\partial x}, \frac{\partial \rho}{\partial y} \right) \quad (16)$$

By computing the gradients of the process field φ , we can obtain the local build orientation $\mathbf{b}$:

$$\mathbf{b} = \left(\frac{\partial \varphi}{\partial x}, \frac{\partial \varphi}{\partial y} \right) \quad (17)$$

Then, the cosine of the overhang angle θ can be evaluated as:

$$\cos \theta = \frac{\mathbf{b} \cdot \mathbf{g}}{\|\mathbf{b}\| \cdot \|\mathbf{g}\|} \quad (18)$$

The overhang surface is defined as the boundary where all the overhang angle θ is larger than a specific value θ_0 , which can be described as:

$$\cos \theta < \cos \theta_0 \quad (19)$$

To better integrate with the optimization model, the following variable χ is defined to identify the overhang violation:

$$\chi = \mathbf{b} \cdot \mathbf{g} - \|\mathbf{b}\| \cdot \|\mathbf{g}\| \cdot \cos \theta_0 \quad (20)$$

Then, the field χ will be further projected into a 0–1 binary vector through Heaviside projection:

$$\bar{\chi} = \frac{\chi}{1 + e^{-\beta\chi}} \quad (21)$$

where β is still the sharpness control parameter. The purpose of Eq. (21) is to map negative elements in χ (boundary elements satisfying the self-support condition) to 0 and positive elements (boundary elements not satisfying the self-support condition) to 1. Subsequently, based on Eq. (21), we can easily construct a volume fraction constraint:

$$\frac{1}{N_E} \sum_{e \in \Omega_D} \bar{\chi}_e \leq V_{tol} \quad (22)$$

The primary purpose of Eq. (22) is to minimize the number of boundary elements that fail to satisfy the self-support condition, thereby achieving the self-support effect of the overall structure. To avoid potential numerical issues, V_{tol} needs to be a very small value (but still greater than 0).

It is important to note that methods relying on boundary directions often encounter numerical issues. One typical problem is the so-called "water droplet" phenomenon [54,55,56,57]. This issue arises from a conflict between the objective function and the self-support constraint. Local geometries, such as droplet tips that support upper structures, cannot be accurately captured by the algorithm as a suspended non-self-support feature. These droplets do not substantially affect the overall mechanical efficiency of the structure, and consequently, large overhangs tend to have water droplets to ensure their self-support satisfaction.

The solution to such an issue is relatively straightforward: we simply enhance the range of elements that are included in the support condition evaluation during the early optimization process. Recalling Eq. (16), obtaining $\mathbf{g}$ requires gradient calculations. Similar to the work in [73], we employ a convolution-based approach for gradient evaluation and importantly, the convolution kernel size is adaptively changed. During early optimization, we apply a large convolution kernel for filtering, magnifying the non-self-support effect of the droplet tip and thus, eliminating the unwanted droplet features. In later iterations, the kernel size is gradually reduced to an appropriate value for accurate boundary gradient calculation.

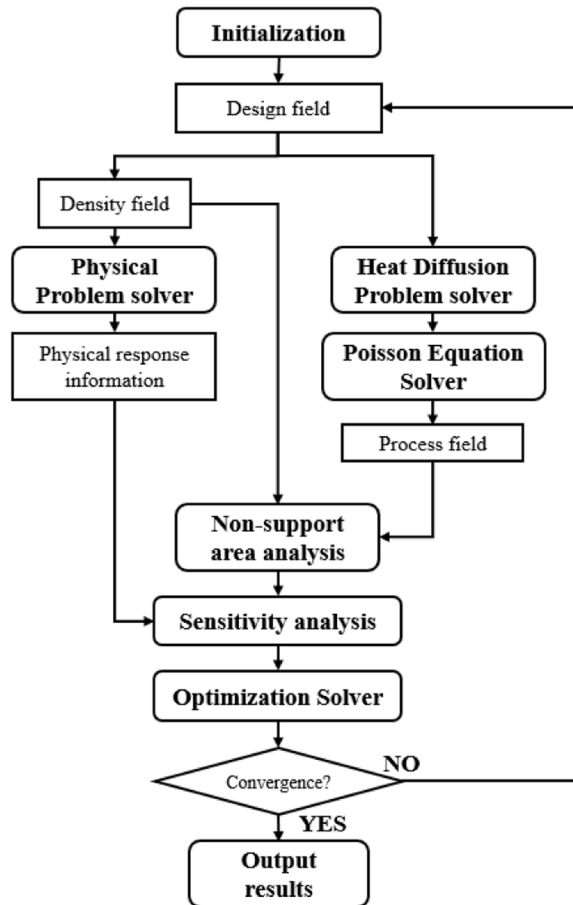


Fig. 9. The flowchart of the proposed method.

4. The concurrent optimization framework

For optimization, we are attempting to find a suitable structure coupled with a suitable process plan, thereby achieving the optimized design in structural performance while satisfying the self-support request for multi-axis AM. Therefore, a method of concurrent structural-process optimization will be developed and detailed in the following.

4.1. The concurrent optimization model

For structural design, our primary concern is to achieve an optimal structural performance under the constraint of structural volume fraction, while ensuring the obtained structure being self-support. The structural topology optimization takes the classic formulation which needs no further explanation. The principle of optimizing the pseudo-density field τ is to minimize the unsupported overhangs by tuning the local build directions. The pseudo-density field is concerned with the thermal diffusivity distribution, that can employ any chaotic distribution, as long as its resulting process field is sufficiently favorable to satisfy the manufacturing constraints. The pseudo-density map represents a ‘structure’ which determines the heat transfer paths, and therefore, we can treat the process optimization problem similarly as a structural optimization problem. In conclusion, the concurrent optimization model can be expressed as follows:

$$\left\{ \begin{array}{l} \text{find : } \mu, \tau \\ \text{minimize : } C_r(\mu) + L_0(\tau) \\ \text{subject to : } \left\{ \begin{array}{l} \mathbf{KU} = \mathbf{F} \quad (23-1) \\ \mathbf{K}_{th}\mathbf{T} = 0 \quad (23-2) \\ \mathbf{K}_{po}\boldsymbol{\varphi} = \mathbf{Q}_{po}(\mathbf{T}) \quad (23-3) \\ \frac{\sum_{e=1}^{N_E} \rho_e}{N_E} \leq V_f \quad (23-4) \\ \frac{\sum_{e=1}^{N_E} \bar{\chi}_e}{N_E} \leq V_{tol} \quad (23-5) \\ \mu_{min} \leq \mu_e \leq 1, \forall e \quad (23-6) \\ \tau_{min} \leq \tau_e \leq 1, \forall e \quad (23-7) \end{array} \right. \end{array} \right. \quad (23)$$

In this context, $C_r(\mu)$ represents the objective function that denotes any measure of structural performance (such as stiffness, maximum stress, maximum deformation, etc.). $L_0(\tau) = \sum_{e \in \Omega_0} \bar{\chi}_e$, is developed by summing up the support condition indication variables, aiming to minimize the number of unsupported elements. Eq. (23–1), Eq. (23–2), and Eq. (23–3) correspond to the control equation system of the physical problem, the heat diffusion, and the Poisson equation problem. Eq. (23–4) represents the volume fraction constraint of the structure, and V_f is the user defined volume fraction upper limit. Eq. (23–5) signifies the self-supporting constraint as mentioned in [Section 3.2](#). μ_{min} and τ_{min} are set as 1.0×10^{-9} , for avoiding matrix singularity during solving the linear system.

4.2. Implementation of the concurrent optimization

The two design variable fields are tightly bonded, for which the second term in objective function can facilitate the satisfaction of the self-support constraint in Eq. (23–5), through which the structural evolution can be less perturbed by this constraint and its feasible design space can be greatly expanded. In this work, the optimization model is solved with the Method of Moving Asymptotes (MMA), and the corresponding critical sensitivity analysis is summarized in [Appendix A](#).

The procedures of the proposed optimization process are given in [Fig. 9](#), and the whole procedures could be described as follows:

STEP. 1 Preparing the necessary information or data for the optimization (the initial density variables for structure design and process planning, FEM mesh, matrixes, MMA solver data and so forth).

STEP. 2 Solving the heat diffusion equation as introduced in [Section 3.1](#) to obtain the temperature distribution $\mathbf{T}$.

- STEP. 3** Processing the temperature distribution T obtained in STEP. 2, and then solving Poisson equation as introduced in Section 3.2 to obtain the process field ϕ .
- STEP. 4** Based on the process field ϕ and density field ρ , following the method described in section 4 to obtain the non-self-support area $\bar{\chi}$.
- STEP. 5** Following the content at the Sections 2–4 to establish the concurrent optimization model.
- STEP. 6** Adopting the adjoint method to obtain the sensitivities on the design variables, and then, using MMA solver to update them.
- STEP.7** If the result satisfied the convergence or stop criteria, the optimization stops; otherwise, it goes back to STEP. 2.

5. Numerical examples and discussion

In this section, we will use several 2D and 3D numerical examples to verify the effectiveness of our algorithm. The 2D cases use 4-node first-order rectangular elements, while in the 3D cases, 8-node first-order cubic elements are utilized. For the 2D cases, unless otherwise specified, R_1 is set to 4 times the element width, and $R_2 = 2R_1$. For the 3D case, R_1 is set to 8 times the element width, and $R_2 = 2R_1$ also. For the gradient kernel size, in the 2D cases, the initial size is set to 15 and reduces by 2 in every 20 iterations until it reaches 3. For the 3D cases, the initial size is set to 11 and reduces by 2 for every 30 iterations until it reaches 3. For the projection parameters, the β is initially set to 2 and double in every 30 iterations until it reaches 32. In both 2D and 3D process optimizations, Γ_{start} is set with a dimensionless temperature of 1, and Γ_{end} with a dimensionless temperature of 0. Finally, the move limit in MMA is set to 0.2. The optimization process terminates when the objective function changes by less than 0.1% over five successive iterations while all constraints are satisfied, or after a maximum of 300 iterations.

5.1. 2-D examples

5.1.1. Short cantilever beam design

The first 2D example is designing a short cantilever part with domain dimensions of $L=200\text{mm}$ and $H=100\text{mm}$, as depicted in Fig. 10(a). A mesh consisting of 100×200 square elements with dimensions of $1\text{mm} \times 1\text{mm}$ is utilized to discretize the design domain. The boundary condition for structural optimization is also illustrated in Fig. 10(a). Specifically, the left boundary is fixed, and a vertical downward force of 1N is applied to the left bottom corner. For process optimization, the blue and purple boundaries are the Γ_{start} and Γ_{end} , respectively. In this example, the optimization objective is to maximize structural stiffness, with a self-support inclination threshold of 45° . The volume fraction constraint is set to 45%. A pseudo elastic isotropic material is adopted in this case, where the Elastic modulus is 1 MPa, and Poisson ratio is 0.3.

Fig. 10(b) displays the topologically optimized structure and its slicing result. This structure exhibits some suspended overhang features that require support structures to maintain stability for traditional 3-axis AM; instead, they turn to self-support because of the keep-rotating printing direction. Additionally, by observing the sliced layers, we find them to be uniform and continuous, appearing no large-curvature bending. This slicing result is advantageous for practicing multi-axis AM. In Fig. 10(c), several important scalar fields are presented.

The convergence information recorded in Fig. 11 shows the historical data of the optimization process. The optimization program was terminated after 300 iterations. The objective function value approached stability in the final iterations. Both the volume fraction

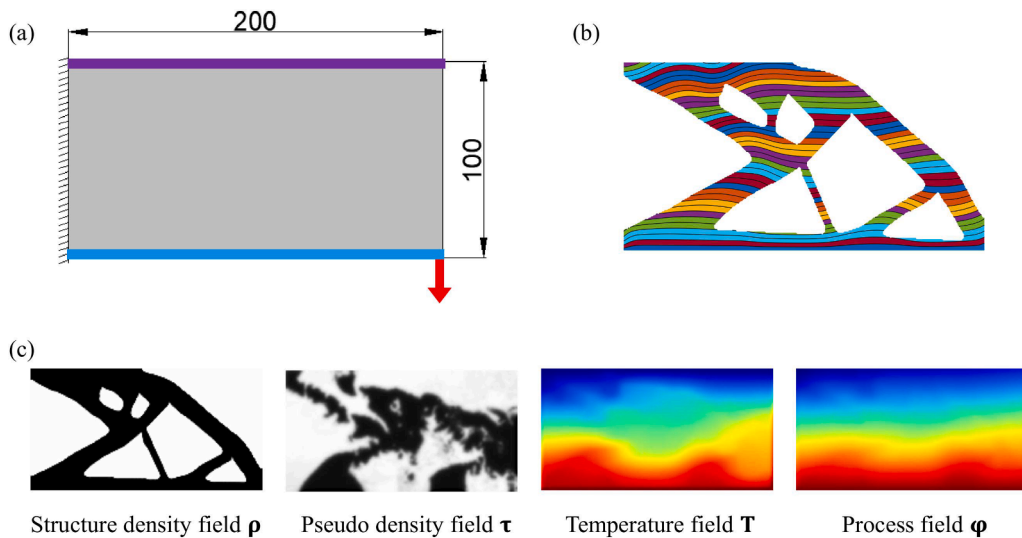


Fig. 10. The boundary condition of the 2D short cantilever example (a); the optimized multi-axis AM design (containing both structure and slicing information); the different colors indicate different layers (b); The important scalar fields involved in this case (c).

constraint and the self-support constraint were successfully satisfied. It was observed that during the initial half of the optimization iterations, all curves exhibited significant oscillations. These fluctuations were attributed to periodic updates of several Heaviside projection parameters and changes in kernel dimension size as per the operation. However, these oscillations diminished during the latter half of iterations, indicating the attainment of a stable structural topology and slicing pattern. Therefore, it can be claimed that the algorithm converges steadily.

5.1.2. Bridge structure design

The second 2D example designs a bridge structure, with domain dimensions defined by $L=300\text{mm}$ and $H=100\text{mm}$, referring to Fig. 12(a). A mesh with 300×100 square elements of size $1\text{mm} \times 1\text{mm}$ discretizes the design domain. Regarding structural optimization, the lower left and right feet of the domain are fixed, and a set of uniform downward forces with the total of 1N are applied to the upper boundary of the design domain. For process optimization, the boundary conditions are consistent with those described in Section 5.1.1. In this example, the optimization objective remains the maximization of structural stiffness, still with the overhang self-support threshold of 45° . The volume fraction upper limit is set to 30%.

Fig. 12 (b) displays the topologically optimized structure and its corresponding slicing result. The primary reason for selecting this case is that the optimization result from this boundary condition cannot be directly printed using traditional bottom-up 3-axis AM technique without the involvement of a large volume of supports. Even if applied of the planar-layer based printing self-support constraint, since the structure is fixed at the two bottom feet, the large span of the top surface that bears the external load should be provided with sufficient back-ups from its below structures, while these back-up structures contribute little to the stiffness enhancement. This contradiction makes it difficult to converge for traditional self-supporting algorithms.

The structural compliance of the optimized bridge is 39.754J . The fabrication sequences corresponding to the optimized process fields are shown in Fig. 13. For the purpose of visualization, we divide the part into 40 layers, with each layer thickness around 2.5mm . A magnified view of the printing is shown at the left side of Fig. 13, wherein the printing reference direction $\vec{b}$ and the boundary normal $\vec{g}$ has the including angle below 45° degrees, ensuring a qualified self-support printing.

5.2. 3-D examples

5.2.1. Bracket structure design

The first 3D example works on a bracket part, as depicted in Fig. 14. The geometric sizes and boundary conditions are defined in the same figure. Due to the symmetry of the structure, only a half is optimized (the green plane is the symmetry plane). The design domain is discretized with a mesh including $160 \times 80 \times 32$ cubic elements. The volume fraction upper limit is set to 20%. For the process optimization boundary conditions, Γ_{start} and Γ_{end} are defined at the bottom and top surfaces of the design domain, respectively.

The final topology optimization result is shown in Fig. 15(a). Due to the involvement of multi-axis printing, several large overhang geometries relative to the horizontal platform (traditionally non-self-support) appear in the final design. The final structural compliance is 330.886J . Fig. 15(b) illustrates the topologically optimized structure along with its associated curved layer slicing result. Moreover, two zoomed-in views are given in Fig. 15(b) to demonstrate the smoothness, continuity and uniformity of the layer generation. Finally, the obtained result is re-meshed and then import into a commercial software (COMSOL 6.2) to simulate the nodal displacement, whose results are shown in Fig. 15(c).

5.2.2. Heatsink structure design

The second 3D example is the heatsink problem as shown in Fig. 16, whose sizes are defined with $L = 128\text{mm}$, $W = 128\text{mm}$, $H = 96\text{mm}$. A mesh with $128 \times 128 \times 96$ cubic elements of size $1\text{mm} \times 1\text{mm} \times 1\text{mm}$ is employed to discretize the design domain. The volume fraction constraint has the upper limit of 20%. Again, a pseudo-isotropic material is adopted in this case, the heat

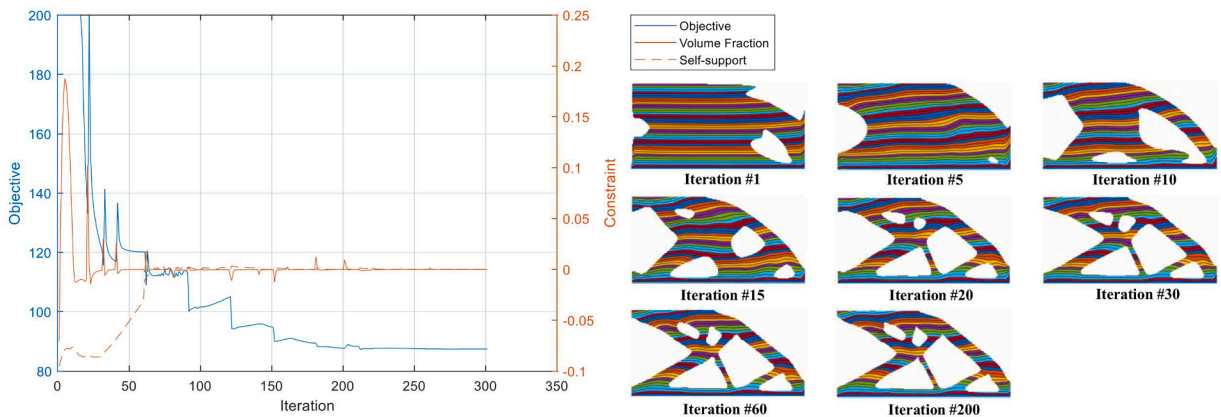


Fig. 11. The historical data of convergence information.

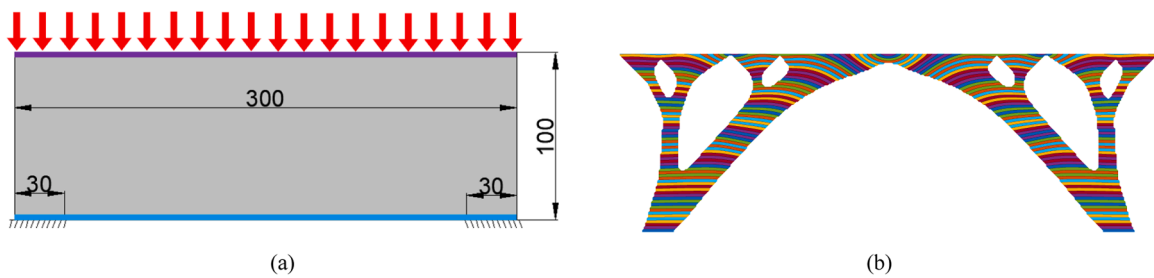


Fig. 12. Illustration of the bridge structure optimization: (a) The boundary condition; (b) The optimized multi-axis AM design (containing both structure and slicing results).

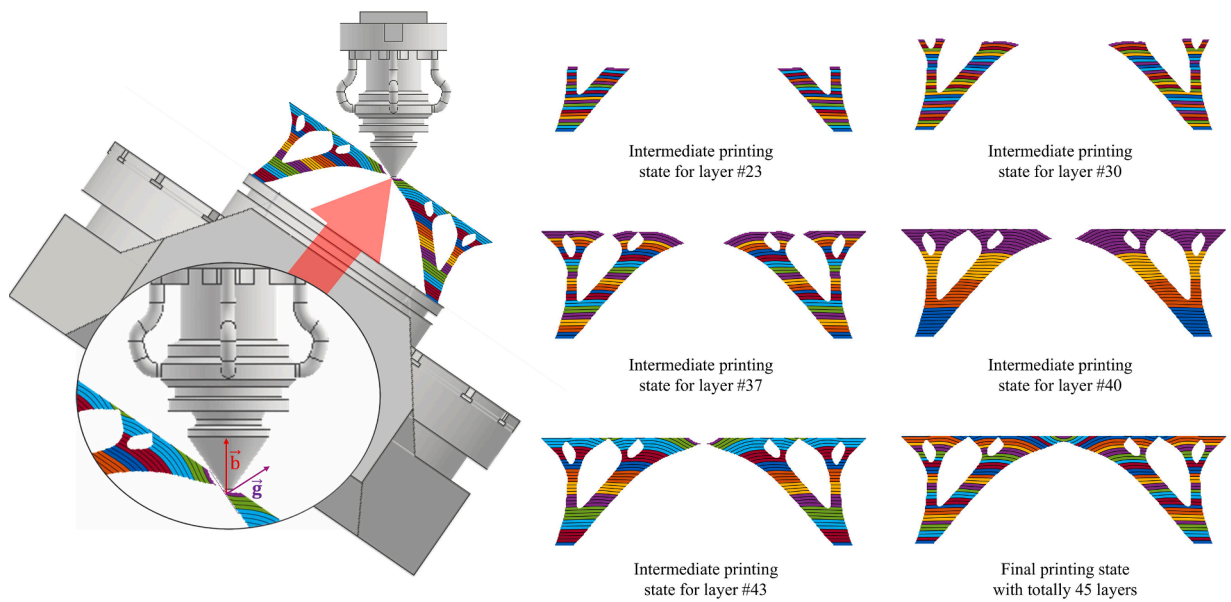


Fig. 13. Illustration of the designated process plan for multi-axis printing.

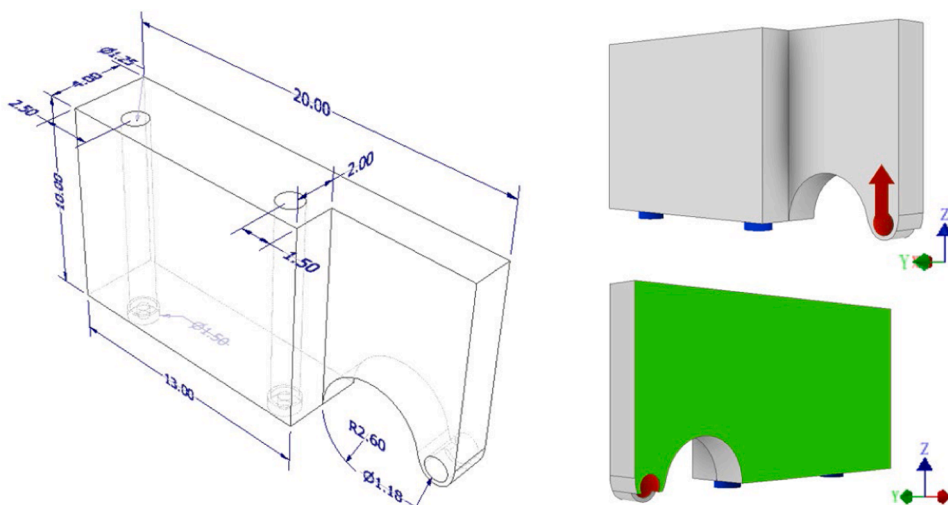


Fig. 14. The boundary condition for the bracket structure (only the back half is shown for being symmetric).

conductivity factor for the solid material is $1 \text{ W}/(\text{m} \cdot \text{K})$, and a uniform heat load with $10 \text{ W}/\text{m}^2$ is applied in the design domain. For the process optimization boundary conditions, Γ_{start} and Γ_{end} are defined at the bottom and top surfaces of the design domain, respectively.

The final optimized heat sink structure is shown in Fig. 17(a). The final so-called thermal compliance is around 330.886J . By examining the final result, we see that under the current printing boundary condition, the materials at the bottom of the design domain connects to the printing platform. As the height increases, overhang structures relative to the horizontal platform emerge. Fig. 17(b) illustrates the topologically optimized structure along with its associated slicing layers, and two zoomed-in views are presented in Fig. 17(b) to demonstrate the smoothness, continuity and uniformity of the slicing result. Finally, the obtained result is re-meshed and then import into COMSOL 6.2 to simulate the nodal displacement, whose results are shown in Fig. 15(c). Two zoomed views of the mesh model for the bottom boundary are shown in left, and the corresponding maximum temperature within the design domain is 469K .

Theoretically, this design does not fully demonstrate the advantages of the proposed algorithm because, considering manufacturing constraints, the growing direction of the sliced layers does not align perfectly with the material distribution direction, and consequently, self-support constraints still impose significant influences on the structural evolution. Therefore, setting the appropriate process optimization boundary conditions is crucial. It is necessary to analyze the boundary condition setup by accounting on the actual manufacturing scenarios and aligning them with the boundary conditions used for structural optimization. This topic will be discussed further in the next subsection.

5.3. Discussion

5.3.1. The comparison amongst different optimization schemes from the performance perspective

Firstly, the comparison amongst different optimization schemes is investigated. The short cantilevers in 2D and 3D, and three optimization schemes (TO-OPT, Planar-layer-OPT, and Multi-Axis-OPT) are considered. For the scheme named TO-OPT, it adopts the conventional topology optimization method, where only the issue about minimum length scale control [74] is considered. Then, for Planar-layer-OPT, it means self-support design while involving the minimum length scale control, and the method in [48] is adopted here for the purpose of comparison. Finally, Multi-Axis-OPT represents our proposed method, and Γ_{start} and Γ_{end} are applied to the bottom and top surfaces of the design domain, respectively. The various optimization schemes are detailed in Table 1.

Fig. 18 displays the optimized structures found from the three optimization schemes mentioned in Table 1. In the case of TO-OPT, the geometric freeform is the highest among the three schemes since it only considers the minimum length scale size. However, as a consequence, many overhang features appear that cannot be printed by the 3-axis 3D printer without supports. 3-Axis-OPT considers both the self-supporting constraint and the minimum length scale control. As a result, its geometry is strictly constrained to follow the 45-degree inclination, but tremendous materials are distributed that do not obviously contribute to enhancing structural performance. The geometric freedom of Multi-Axis-OPT lies between Planar-layer-OPT and TO-OPT. Compared to Planar-layer-OPT, it allows the appearance of large overhang features. Its structural topology remains the same as TO-OPT, with only changes in shape.

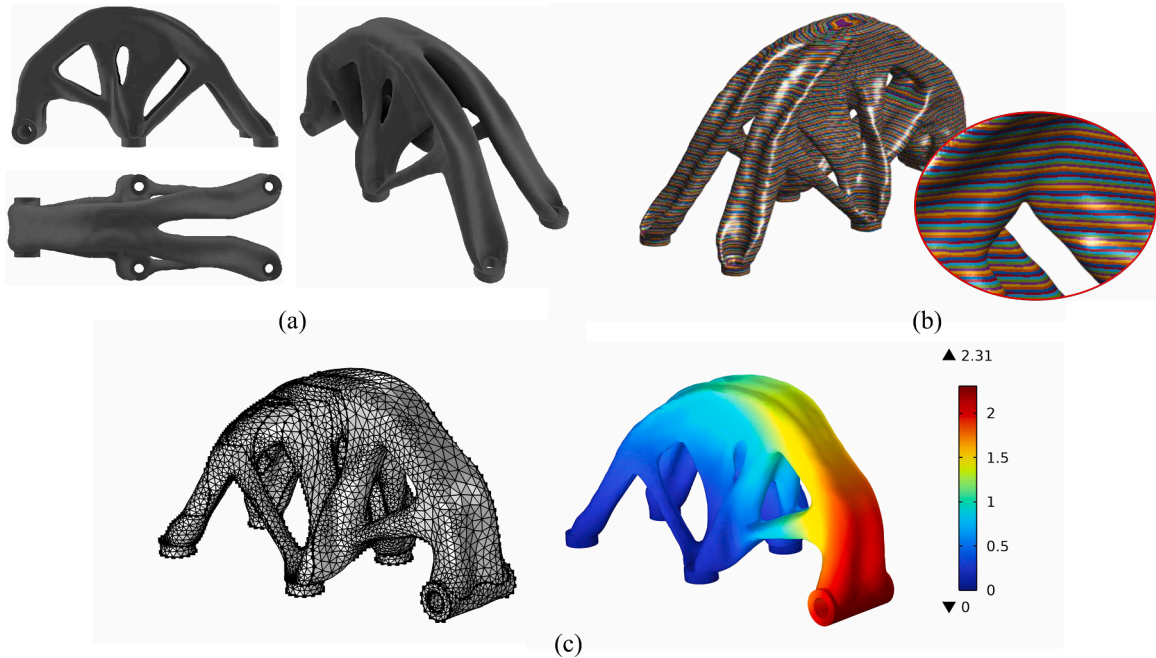


Fig. 15. The optimized bracket part for multi-axis 3D printing, containing both the structure design (a), slicing result (b), and the simulation result (c): where the left is the mesh model used for simulation, and the right is nodal displacement distribution (unit: mm).

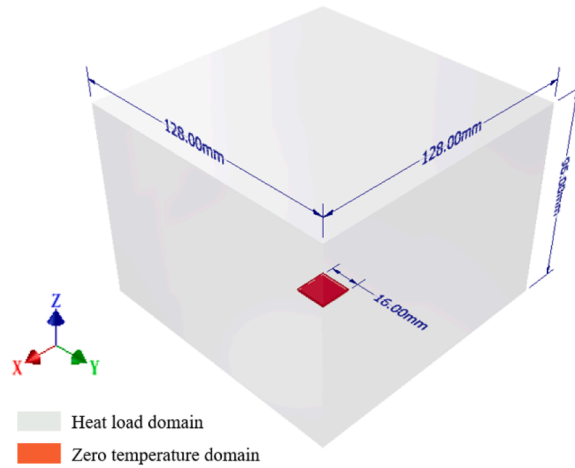


Fig. 16. The boundary condition for the 3D heat sink example.

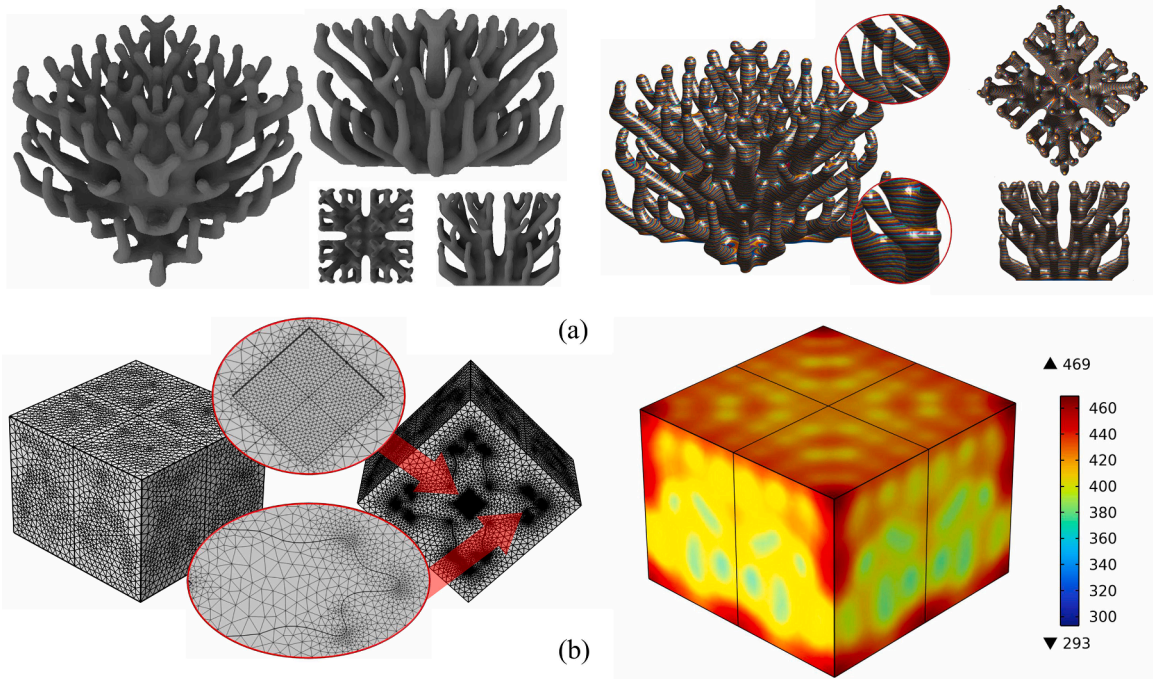


Fig. 17. The optimized multi-axis AM design for the 3D heat sink (showing both the structure (a) and slicing result (b)), and the simulation result (c): where the left is the mesh model used for simulation, and the right is nodal temperature distribution (unit: K).

Fig. 19 displays the 3D results by the three optimization strategies. Different from the phenomenon observed in the 2D results, the structural topology from Multi-Axis-OPT does not keep consistent with TO-OPT.

The structural performances from these three optimization schemes are concluded in Fig. 20. For 2D case, the structural performance of TO-OPT (81.165J) is the best, followed by Multi-Axis-OPT (84.566J), while Planar-layer-OPT performs the worst (91.207J). Compared to Planar-layer-OPT, the performance of Multi-Axis-OPT exhibits an improvement of approximately 6.4%; For 3D case, the structural performance of TO-OPT (322.774J) is the best, followed by Multi-Axis-OPT (330.866J), while Planar-layer-OPT performs the worst (346.788J). Compared to Planar-layer-OPT, the performance of Multi-Axis-OPT exhibits a slight improvement of approximately 4.7%.

5.3.2. Manufacturability analysis compared with the direct slicing result of traditionally non-constrained topological designs

Second, regarding the necessity of concurrent optimization from a manufacturing perspective, we provide the following design example, comparing the TO-OPT and Multi-Axis-OPT methods. A more professional and advanced slicer was used to generate the slices

Table 1

The summary of the optimization schemes.

Scheme name	Minimum length scale control	Self-support constraint	Multi-axis printing optimization	Reference
TO-OPT	✓	×	×	[74]
Planar-layer-OPT	✓	✓	×	[48]
Multi-Axis-OPT	✓	✓	✓	-

for both structures to investigate their manufacturability. In this case, the design considers a 3D cantilever beam with Dirichlet boundary conditions applied for the density filter. In this example, we employed an $80 \times 40 \times 32$ voxel mesh, with the structure dimensions defined as Length = 80, Width = 32, and Height = 40 (dimensionless). The material properties are consistent with those used in previous content. Fig. 21(a) shows the results obtained from TO-OPT, and Fig. 21(b) shows the results from our algorithm. In our algorithm, the bottom plane is defined as the starting plane for printing (i.e., the $z = 0$ xy-plane), and the corresponding slicing results are shown in Fig. 21(c). The structural performance of the two optimization strategies is as follows: the TO-OPT result achieves a compliance value of 147 (dimensionless), while our algorithm achieves a compliance value of 153 (dimensionless). Comparatively, the structural performance of our algorithm is 4% worse than that of the traditional topology optimization.

Then, we used the S³ slicer [71] with the open-source code (https://github.com/zhangty019/S3_DeformFDM) to perform slicing of both structures. The printing starting point for both cases was defined as the $z = 0$ xy-plane, and the results are shown in Fig. 22. According to the slicing results generated by S³ slicer, the fabrication of horizontally connected beams requires a step-by-step approach from both ends toward the center. However, when the nozzle direction aligns with the layer normal direction, collisions between the laser head and the already built structure are unavoidable, as illustrated in Fig. 23(a) and (b).

By dynamically adjusting the laser head direction (so that it does not completely align with the layer normal direction but is slightly deflected), the aforementioned collision issue can be alleviated to some extent. For example, in Fig. 23(c), the laser head direction is tilted to deviate the laser head body from the structure, and geometrically, this change takes effect in avoiding collision. However, for multi-axis DED process, the tilted laser head leads to instable melt pool formation near the boundaries. The high-speed gas ejection from the nozzle can cause portions of the melt pool to flow toward outside the structure; see Fig. 23(c) for the illustration. Additionally, since the melt pool is reshaped and deviated from the powder ejection focus, a large amount of powders, unmelted or partially melted, would splatter onto the printing platform, causing significant powder waste but also damaging the quality of already built part surface. This phenomenon and its adverse effects have been thoroughly reported in a recent study [81]. In contrast, referring to Fig. 23(b), it is suggested to keep the laser head direction aligned with the layer normal direction or slightly tilted to eject powders toward the inner structure, through which enables a more stable and uniform melt pool formation. Both the material utilization rate and the manufacturing quality can be guaranteed in this way.

Although the geometric configuration of our structure is less efficient in terms of structural performance compared to the geometry obtained from traditional topology optimization, it achieves a better-balanced optimization effect. By altering the overall topology of the structure, it avoids the occurrence of challenging-to-print geometric features, such as the horizontally connected beams, while only a limited compromise in structural performance has been identified.

At the end, we want to emphasize that the manufacturability analysis is performed on multi-axis DED additive manufacturing. If altered to the filament-based FDM process, very different conclusions can be obtained that altering the printing nozzle direction can be effective in avoiding interference while not obviously degrading the printing quality.

5.3.3. The effect of different printing strategies

Subsequently the influence of different printing strategies is investigated. In this section, various printing strategies are implemented by setting different boundary conditions for solving the Eq. (9). The bridge design in 2D and heat sink design in 3D are

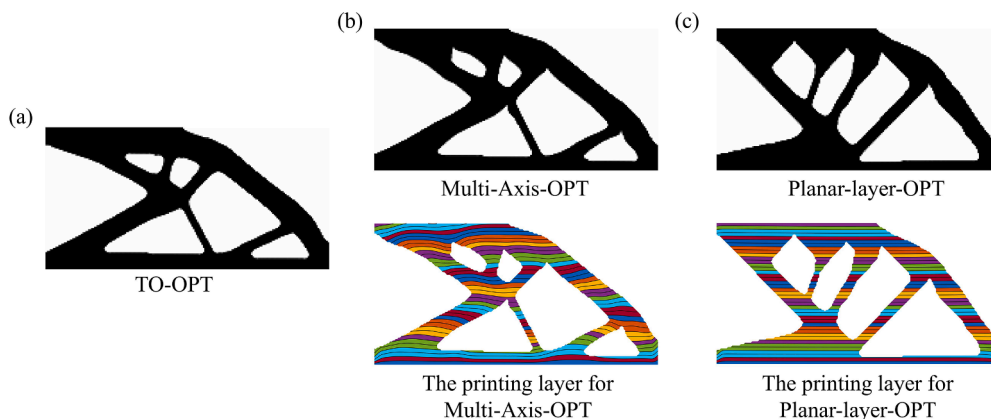


Fig. 18. The optimized 2D short cantilever designs obtained from TO-OPT (a), Multi-Axis-OPT (b), and Planar-layer-OPT (c).

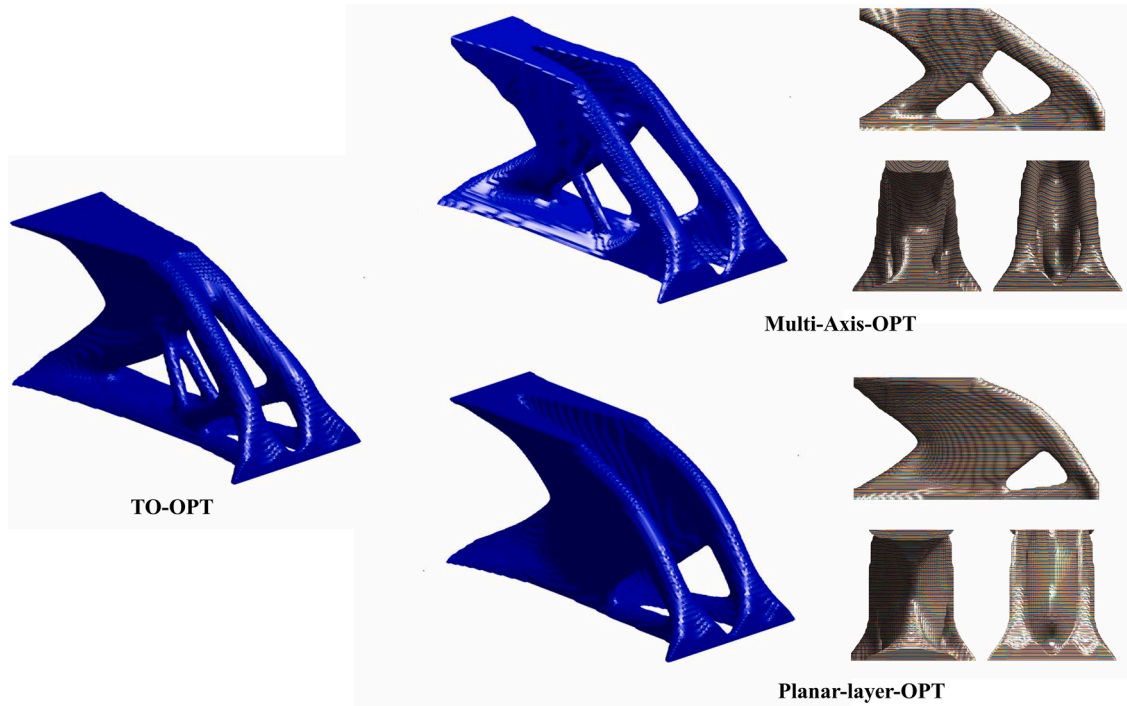


Fig. 19. The optimized 3D short cantilever designs obtained from TO-OPT (a), Multi-Axis-OPT (b), and Planar-layer-OPT (c).

considered. In the 2D bridge design, two printing strategies are examined, each with its corresponding boundary conditions illustrated in Fig. 24. The structures are applied of two Dirichlet boundaries: the blue boundary given a high temperature, while the purple boundary set with a low temperature. The shapes of the low-temperature boundaries are different for the two optimization scenarios.

Fig. 24 shows the optimized structures resulted from the two boundary conditions. Their final optimized structural compliances are 38.248J (Fig. 24(a)) and 40.275J (Fig. 24(b)), respectively. It is evident that, the distinctions in process planning setup have an evident impact on the optimization result, regardless of the structural topology or the slicing pattern.

Next, we revisit the 3D heat sink problem. Apart from the process planning boundary condition used in Section 5.2.2, a new boundary condition is adopted here to make it consistent to the structural heat conduction analysis (Fig. 25). The optimized structure and its corresponding slicing result are shown in Fig. 26, with a thermal compliance of 309.129J.

In Fig. 26, the layer growth directions align with the geometry extension orientations owing to the modification to the process planning boundary condition. This observation is very different from the overall upward growth trend of Fig. 17(b). Consequently, the result in Fig. 26 shows fewer structures directly connected to the platform and more structures extending outward along the heat diffusion paths. Clearly, under the current design condition, the obtained structure achieves a better performance (309.129J vs. 330.886J and maximum temperature 446K vs. 469K).

5.3.4. Slicing quality control

As shown in Fig. 2, excessively curved layers can significantly degrade printing quality and cause collisions between the printer head and the already-shaped structure. Therefore, it is necessary to control the curvatures of the sliced layers. Additionally, the uniformity of layer thickness is crucial as well, since large variations in slicing layer thickness can hardly be realized by the current 3D printing techniques. The latest multi-axis printing techniques allow for dynamic control of printing parameters for achieving non-uniform layer thicknesses, but strict restrictions are still imposed on the thickness deviations [79,80]. Therefore, it is essential to reasonably evaluate and control the sliced layers.

It is worth noting that the curvature of the sliced layers and the uniformity of thickness are interrelated. Generally, less curved layers employ a better uniformity in layer thickness. Recalling the proposed process planning method, the differences in thermal conductivity between two media can lead to obviously altered temperature gradients. Therefore, by adjusting the filtering radius R_2 and the minimum thermal conductivity $\kappa_{\min}$, we can effectively control the curvature distribution and thickness uniformity of the printing layers.

Here, we used a typical 2D MMB beam case to investigate this topic. As a benchmark, its boundary condition is well defined and thus will not be repeated. Regarding the layer slicing, the boundary condition is the same as the cases we conducted in Section 5.1.1. Three groups containing a total of 9 cases are used in this investigation. The groups are divided according to different R_2 , and within each group, the cases will be given different $\kappa_{\min}$. Their settings are summarized in Table 2, while other parameters keep the same with previous 2D cases. The optimized structures and corresponding slicing results are summarized in Table 2.

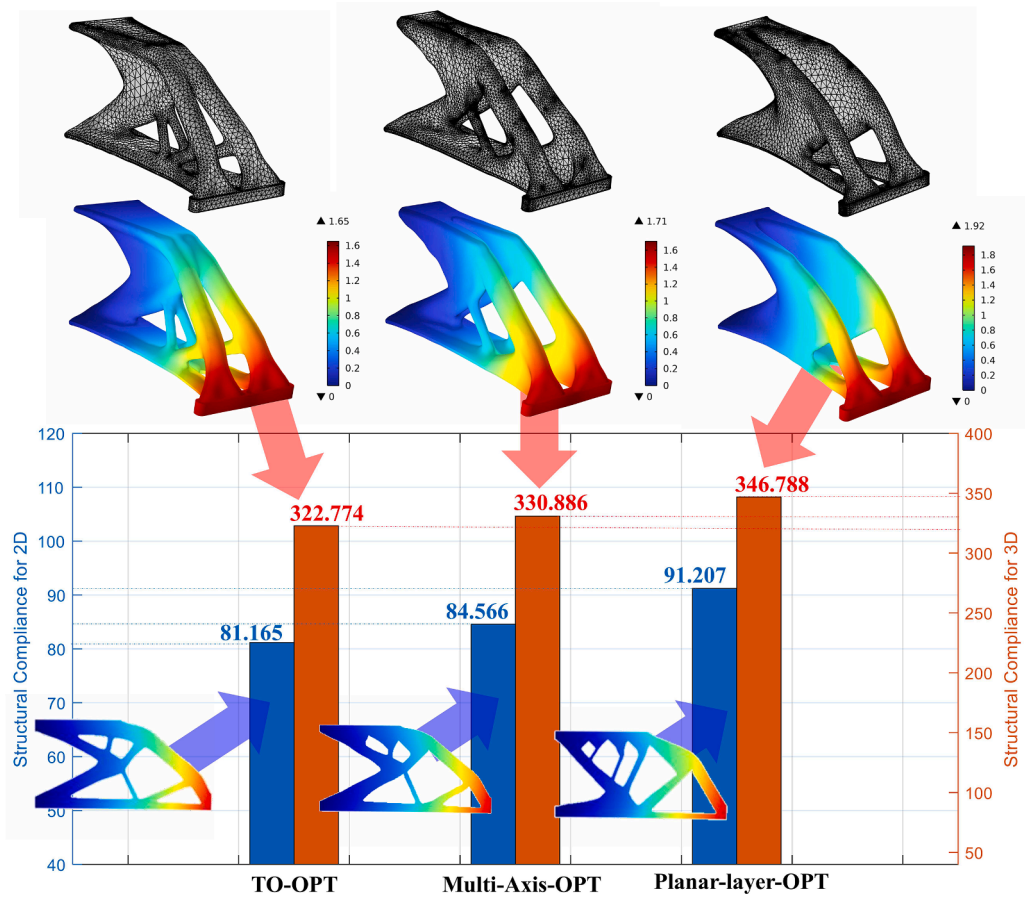


Fig. 20. The structural performance data for the TO-OPT, Multi-Axis-OPT, and Planar-layer-OPT.

5.3.4.1. Layer thickness. Despite employing a series of operations, the solution obtained from Eq. (10) for the process domain cannot guarantee absolute uniformity. Therefore, we need a criterion to determine the levels of uniformity. In this subsection, we adopt an intuitive approach to assess the uniformity in layer thickness, by calculating the variance of the gradient norms of the process field for the entire solid region:

$$T_{\text{var}} = \text{var} \left(\rho \circ \sqrt{\left(\frac{\partial \varphi}{\partial x} \right)^2 + \left(\frac{\partial \varphi}{\partial y} \right)^2} \right) \quad (24)$$

where $\text{var}(\cdot)$ means the variance evaluation process (the variance calculation function in MATLAB). For 3-axis AM with uniformly sliced layers, the variance theoretically equals 0. Definitely, a higher variance value means poorer uniformity of the gradient norms of the process field, and thus indicating a less even slicing result.

In Fig. 27, the optimized structures, sliced layers, and their corresponding thickness variances (T_{var}) under different $\kappa_{\min}$ and R_2 values are summarized. We can see that the thickness variance decreases with larger $\kappa_{\min}$ and R_2 . When $\kappa_{\min}$ is 1.0×10^{-1} , the differences in results among various R_2 values are minimal. However, as $\kappa_{\min}$ decreases, these differences become more pronounced. Additionally, the change rate of T_{var} against $\kappa_{\min}$ varies with different R_2 values. Regarding a smaller R_2 , T_{var} changes more significantly against the spanning $\kappa_{\min}$. Therefore, we can conclude that larger $\kappa_{\min}$ and R_2 are the priorities for selection.

5.3.4.2. Layer curvature. Here, we employ the divergence of the process field as an indirect criterion to evaluate the minimum curvature radius of the layers. Specifically, the normalized direction field of the process field is obtained by:

$$\mathbf{b}_{\text{norm}} = \left(\frac{\frac{\partial \varphi}{\partial x}}{\sqrt{\left(\frac{\partial \varphi}{\partial x} \right)^2 + \left(\frac{\partial \varphi}{\partial y} \right)^2}}, \frac{\frac{\partial \varphi}{\partial y}}{\sqrt{\left(\frac{\partial \varphi}{\partial x} \right)^2 + \left(\frac{\partial \varphi}{\partial y} \right)^2}} \right) \quad (25)$$

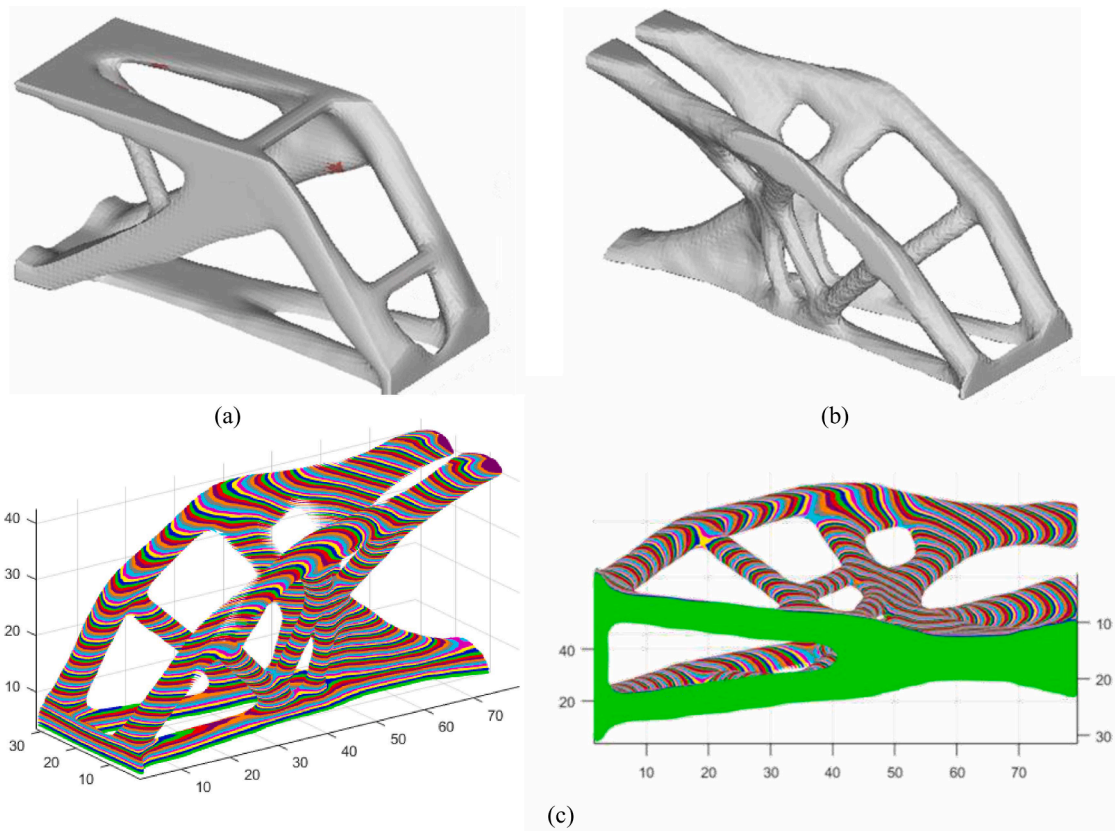


Fig. 21. The optimized structural results for TO-OPT (a), Multi-Axis-OPT (b), and the curved layers created from Multi-Axis-OPT (c).

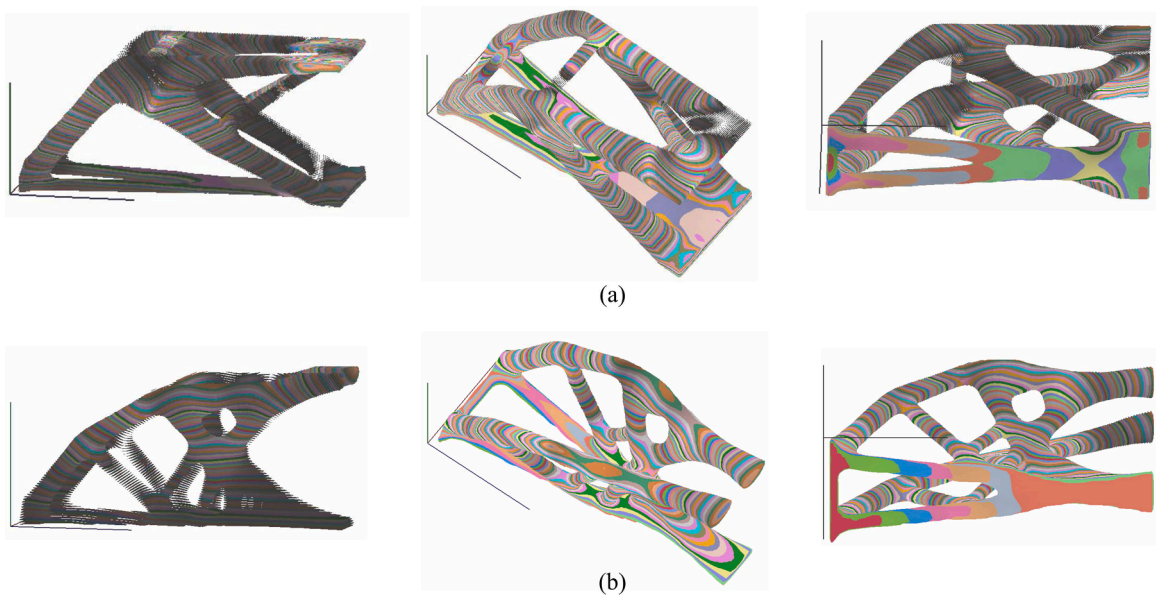


Fig. 22. The curved layers generated by S^3 slicer for designs from (a) TO-OPT and (b) Multi-Axis-OPT.

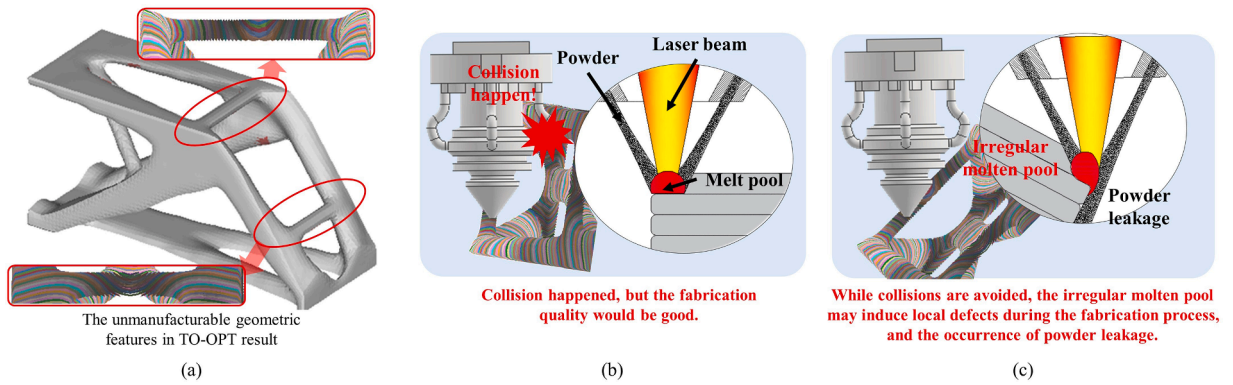


Fig. 23. Manufacturability analysis of the horizontal beams appeared in TO-OPT result: (a) demonstration of the curved layers; (b) better melt pool stability when ensuring the laser head direction aligned with the layer normal; (c) the reshaped melt pool and metal powder leakage due to tilted laser head for avoiding collision.

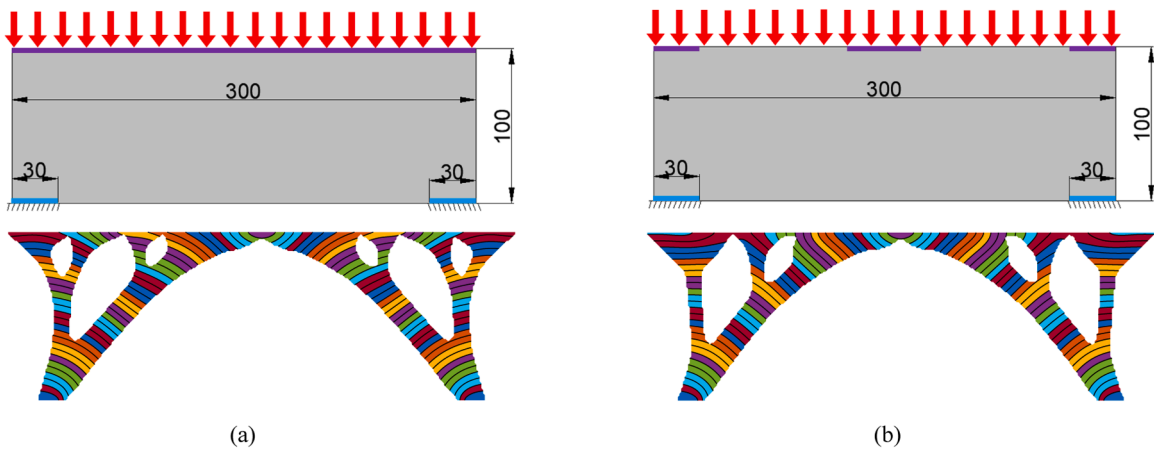


Fig. 24. Illustration of the two process planning boundary conditions and their corresponding optimized structures with layer generation information.

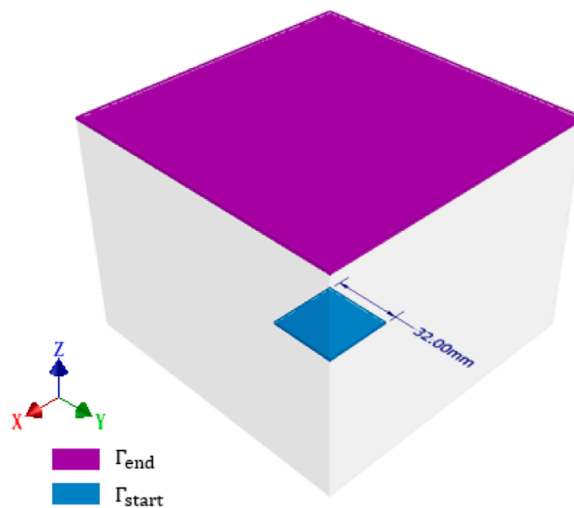


Fig. 25. The new process planning boundary conditions for 3D heat sink.

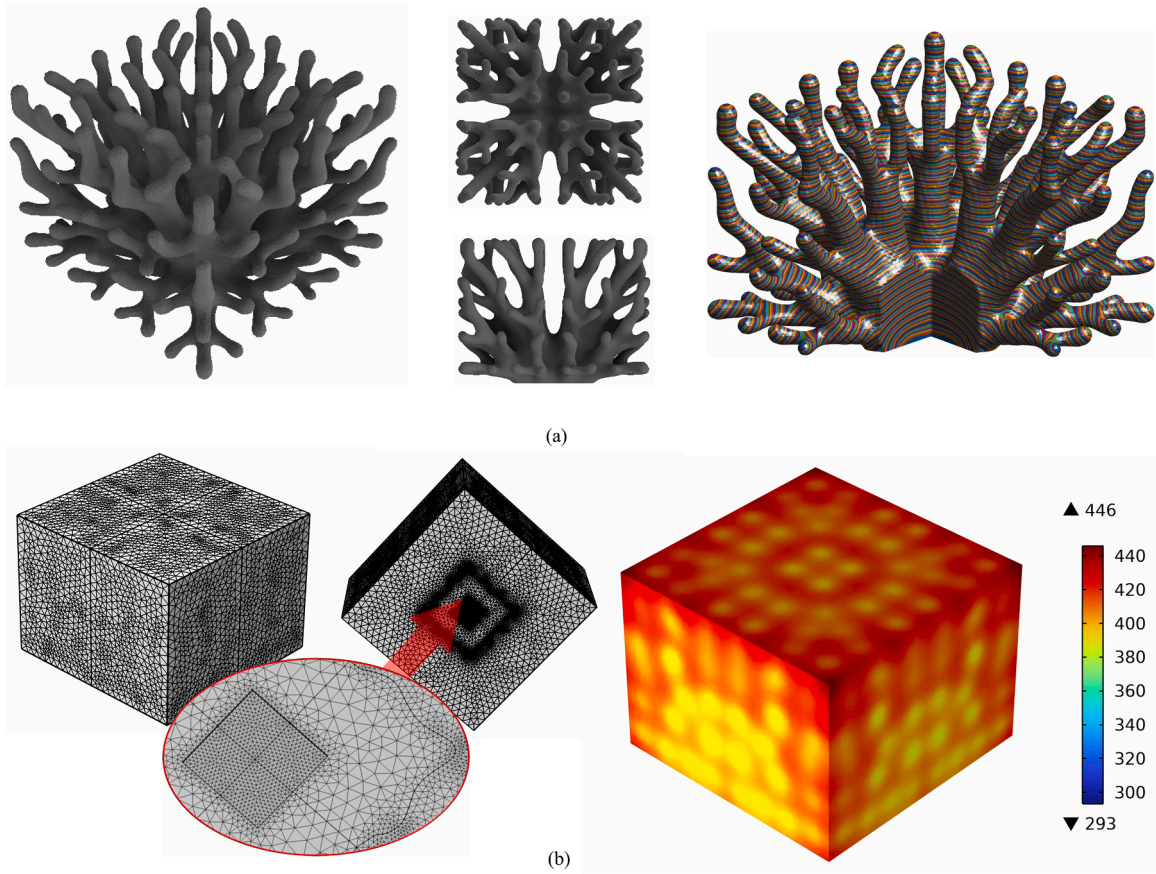


Fig. 26. The optimized 3D heat sink for multi-axis AM with a new boundary condition for process optimization (a), and the simulation result (b): where the left is the mesh model used for simulation, and the right is nodal temperature distribution (unit: K).

Subsequently, the divergence of the process field in e^{th} element is calculated based on Eq. (26):

$$D_{\text{div},e} = \nabla \cdot \mathbf{b}_{\text{norm},e} \quad (26)$$

However, calculating divergence based on individual elements often results in overly localized outcomes, which may not accurately reflect the actual layer curvature. To address this issue, the divergence of the target element will be represented by the weighted average of the divergences of the surrounding elements, denoted as D_{ave} :

$$D_{\text{ave},e} = \frac{\sum_{i \in N_e} D_{\text{div},i}}{\sum_{i \in N_e} 1} \quad (27)$$

N_e is the set of all surrounding voxels with the centroid distances to e^{th} element that are smaller than R_3 , for example:

$$N_e = \{i | \| \mathbf{x}_i - \mathbf{x}_{e\text{AptCommand2016};2} \| \leq R_3 \} \quad (28)$$

where $\mathbf{x}_e$ is the e^{th} element's location, i is the nearby element's index, and $\mathbf{x}_i$ is its location. Recalling that in our case, in the process field, the overall temperature diffusion direction is from bottom to top, so only the maximum positive D_{ave} needs to be concerned:

$$D_{\text{max}} = \max(D_{\text{ave}}) \quad (29)$$

Definitely, a higher D_{ave} means a sharper curve of the printing layer.

Observing Fig. 28, similar to the variance in thickness, the maximum divergence (D_{max}) reduces with larger κ_{min} and R_2 . When κ_{min} is 1E-1, the differences in D_{max} across various R_2 values are the minimal. As κ_{min} decreases, the differences become more pronounced. The change rate in D_{max} against varying κ_{min} also depends on R_2 : a smaller R_2 causes larger variations in D_{max} against the varying κ_{min} . Notably, when R_2 is relatively large (e.g., $R_2=16$), D_{max} shows no significant change with variations in κ_{min} .

Table 2
Summary of the optimization schemes.

$\kappa_{\min}$	Group #		
	1 ($R_2 = R_1 = 4$)	2 ($R_2 = 2R_1 = 8$)	3 ($R_2 = 4R_1 = 16$)
1.0×10^{-1}	 Case-A	 Case-D	 Case-H
1.0×10^{-2}	 Case-B	 Case-E	 Case-I
1.0×10^{-3}	 Case-C	 Case-F	 Case-G

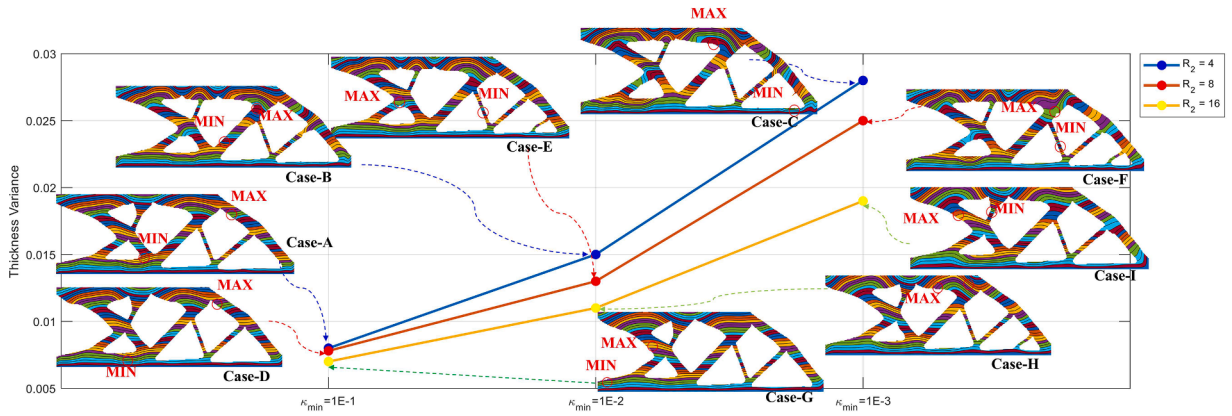


Fig. 27. The variation of T_{var} under different design parameters, along with the corresponding changes in topology and printing paths. The red circles indicate the positions where the maximum and minimum T_{var} values occur.

5.3.4.3. Performance. Finally and most importantly, we investigate the relationship between layer slicing quality and structural performance. We compare the three sets of data from Table 2, with the results plotted in Fig. 29. As observed, the overall structural performance improves if either R_2 or $\kappa_{\min}$ decreases.

Recalling the discussions in the previous two subsections, as the values of R_2 and $\kappa_{\min}$ decrease, the generated process fields become more flexible. As shown in Fig. 29, when both R_2 and $\kappa_{\min}$ are minimized (case C), the resulting structure closely resembles that of conventional topology optimization. However, such process fields lead to poor quality in terms of printing layer curvature and uniformity of thickness. Therefore, in practical applications, it is crucial to carefully set the values of R_2 and $\kappa_{\min}$, balancing between structural performance and layer generation quality.

5.3.5. Perspectives from the design for Computer Aided Manufacturing

As previously mentioned, we ultimately obtain slicing information by solving a Poisson equation. Currently, we also have a

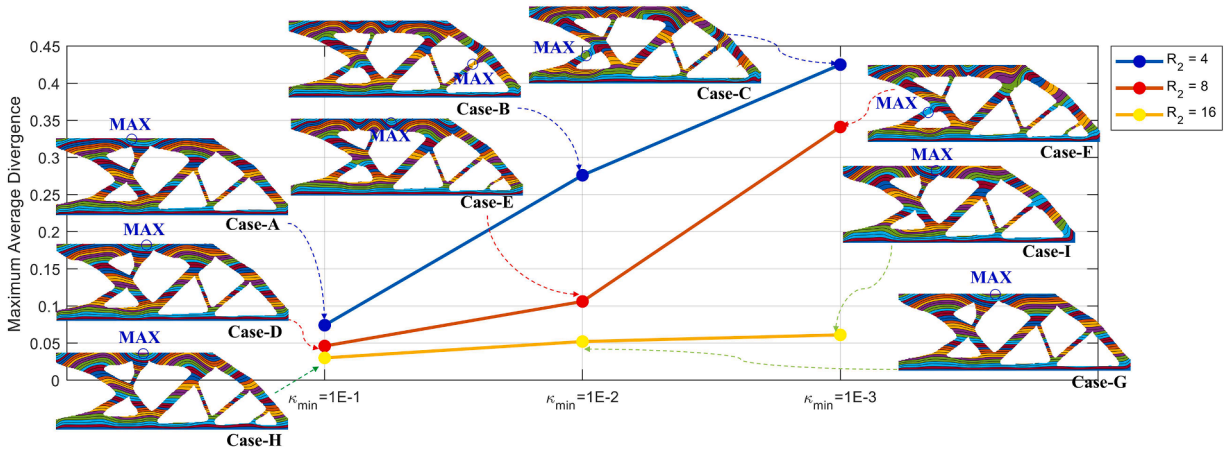


Fig. 28. The variation of $D_{\max}$ under different design parameters, along with the corresponding changes in topology and layer slicing result. The blue circles indicate the positions where the $D_{\max}$ occurs.

systematic approach to achieving an “apple-to-apple” transformation of our optimized result into the execution path code to support design for Computer Aided Manufacturing (CAM). Here, we present the typical example of an optimized heat sink part with an open-topology configuration as the case study. The whole process could be divided into 3 steps:

STEP.1: The optimization process, whose method is the same as we described in Sections 2 to 4, is shown in the top of Fig. 30.

STEP.2: Based on the process optimization result ϕ , we normalized it and then, perform isosurface extraction and adjust the corresponding value to change the height of the isosurface (from 0 to 1). Each isosurface is then converted into an STL model. Similarly, we extract the isosurface of the structural field ρ and convert it into an STL model. Then, we use an in-house STL Boolean operation algorithm to generate each STL slice. The left bottom of Fig. 30 shows the results for randomly two layers, where the black curved surface represents the printing surface extracted from ϕ . Repeat the above steps until all slices are fully segmented. In this case, we divided the model into 175 layers, with an average layer thickness of $0.5 \text{ mm} \pm 0.02 \text{ mm}$. The lower-right portion of Fig. 30 shows the final sliced STL model. Based on this STL model, we then extract the critical boundary node information from each slice and verify it to ensure accuracy.

STEP.3: Based on the information obtained from STEP 2, we can readily use it for path planning and convert it into G-code. Finally, we used CAM software to simulate the paths generated by the G-code to verify its effectiveness, and then proceeded with printing (Fig. 31).

In this process, all critical information for path generation is derived from our design optimization results (ϕ and ρ). We believe this sufficiently demonstrates the strong correlation between our optimized results and the actual path.

5.3.6. Computational cost analysis

Based on the algorithm, the computing time consists of six parts, which is shown in Fig. 32. We utilized the OpenMP library to develop an in-house Multi-Grid Conjugate Gradient (MGCG) solver to address all related PDEs, as well as the OpenMP version of the MMA code, and then used MATLAB’s MEX function to compile them into a .mex file to adopt it within the MATLAB environment. Other

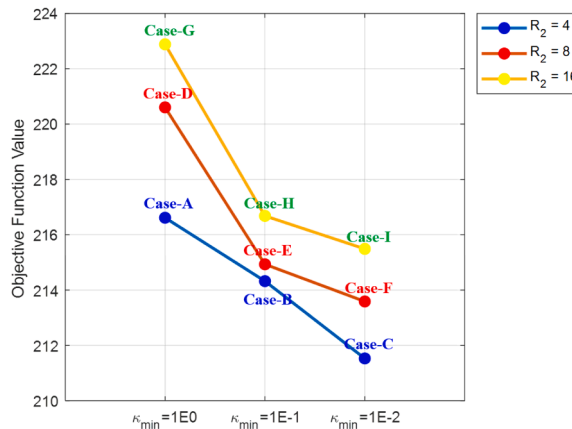


Fig. 29. Structural performance corresponding to the different sets of process planning parameters.

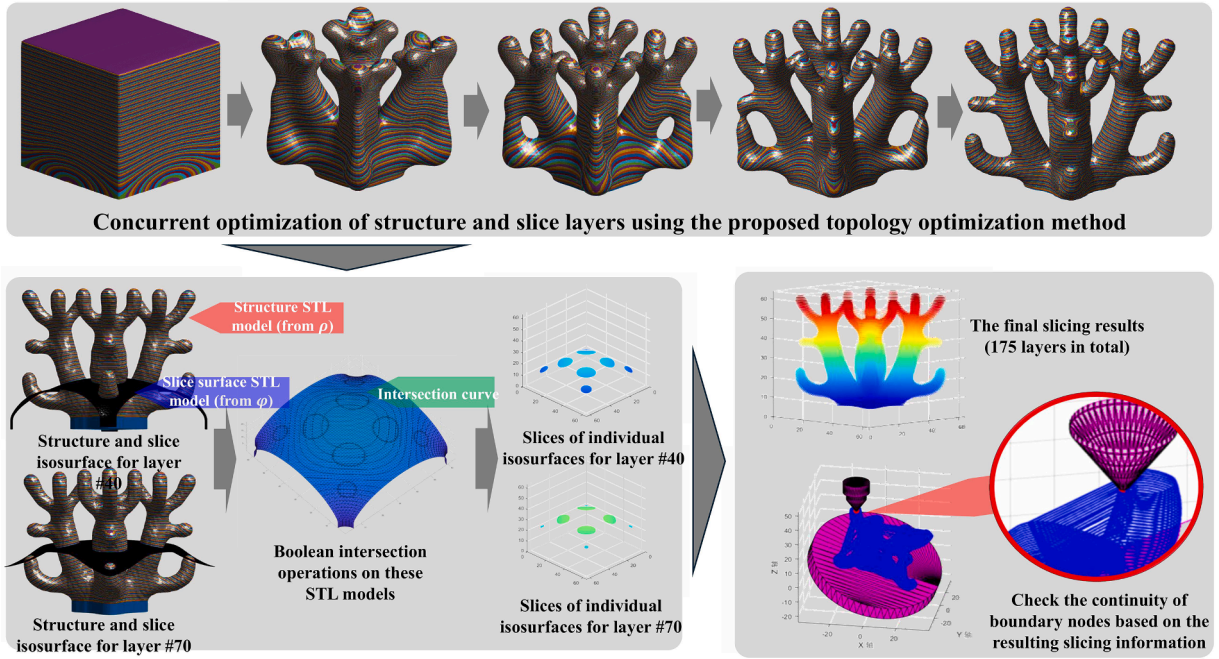


Fig. 30. The workflow for how to convert out results into a real printing path code.

components, such as matrix operations and post-processing, were completed in MATLAB. The hardware resources used are as follows: we utilized a 24-core Intel i7-13,700 CPU with 128 GB of RAM, operating at a base frequency of 2.10 GHz, which can often be overclocked to 4.0 GHz. The MATLAB version is 2023b, and the OpenMP version is 3.7.

The ratios of these six parts to the total time summarized from the different examples are shown in Fig. 33. We can see that, for different cases, the computational components are different. For the case shown in Fig. 14, the total computational time is 4121s. $160 \times 80 \times 32$ cubic elements are involved, and the structure governing equation has 3 degrees of freedom in each node. Therefore, nearly 40% of the computing time is consumed for solving the structure governing equation, while the other parts related to solving PDEs takes around 30%. For the case shown in Fig. 25, the total computational time is 7028s. $128 \times 128 \times 96$ cubic elements are involved, and the structure governing equation has only 1 degree of freedom in each node. Therefore, only 5.5% of the computing time is consumed for STEP.1, while the other parts related to solving PDEs takes around 60%. Specifically, the time for STEP.2 is the most, consisting almost 44%.

6. Conclusion

In this work, we developed a topology optimization algorithm incorporated with the numerically tractable process planning to concurrently design the structure and perform the curved layer slicing for multi-axis additive manufacturing. Self-support constraints

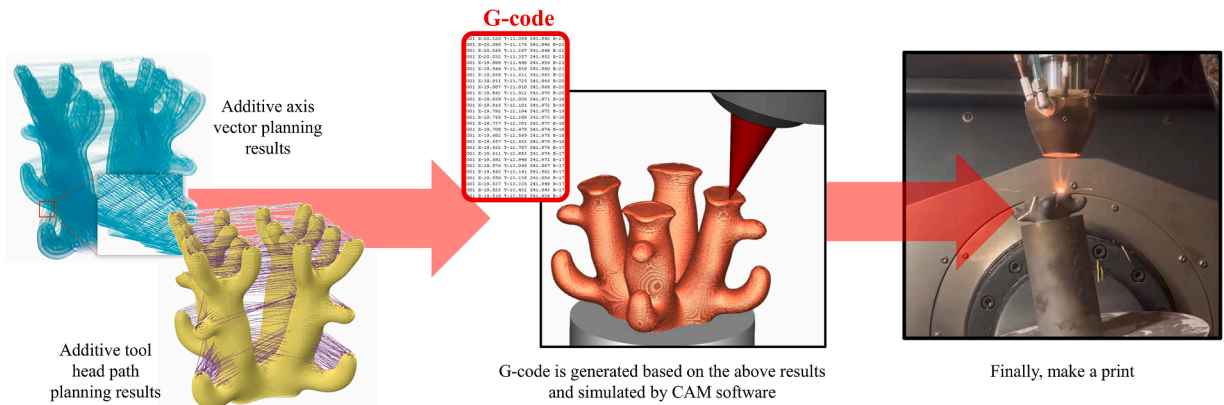
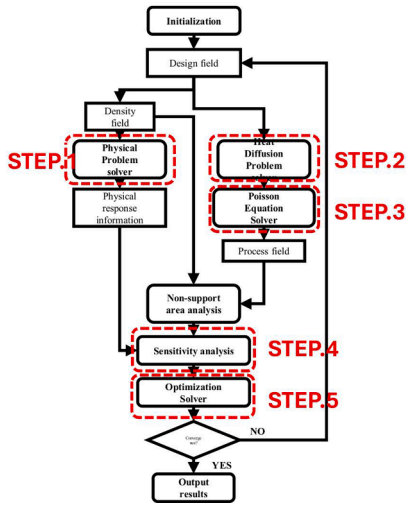


Fig. 31. The validation of result path and manufacturing process.



STEP Number	Description
STEP.1	Solving the structure governing equation (Eq.23-1)
STEP.2	Solving the heat diffusion equation (Eq.23-2)
STEP.3	Solving the Poisson equation (Eq.23-3)
STEP.4	Conducting the sensitivity analysis: solving two adjointed equations (Eq.a9 and Eq.a10)
STEP.5	Adopting the MMA solver
STEP.6	Other parts of the loop

Fig. 32. The computational part description.

are satisfied while their disturbance to the structural evolution has been reduced to minor, leading to optimized structural performances getting close to the conventional non-constrained designs.

Based on our findings, we draw the following conclusions:

1. The self-support issue within the multi-axis AM context has been addressed. Compared to traditional 3-axis printing, the proposed method enables designs with better structural performance while still meeting self-support requirements.
2. The self-support constraints are incorporated into the optimization formulation in the form of explicit volume restriction, which can be easily extended to other physical optimization problems.
3. We have proposed a multi-physics-based slicing optimization model that ensures the continuity and uniformity of the curved layers. This optimization model integrates well within the current mainstream topology optimization frameworks, with all sensitivities being derivable. Therefore, it is straightforward to utilize mature gradient-based solvers for problem-solving.
4. With appropriate parameters, the slices generated by the proposed method are smooth, continuous, and uniform. After post-processing, these process details can be directly applied to subsequent manufacturing operations.

For the methodology, we also have several conclusions:

1. R_2 and $\kappa_{\min}$ are the two key parameters that could affect the slicing quality. In practical applications, it should carefully set the values of R_2 and $\kappa_{\min}$, balancing between structural performance and curved layer quality.
2. Besides, the slicing quality control mentioned in Section 5.3.3 could also be converted into constraints and then, integrated into the proposed algorithm, because all of these evaluation criteria are differentiable. However, to keep the proposed algorithm as simple and understandable as possible, we chose not to implement these constraints at the current stage.

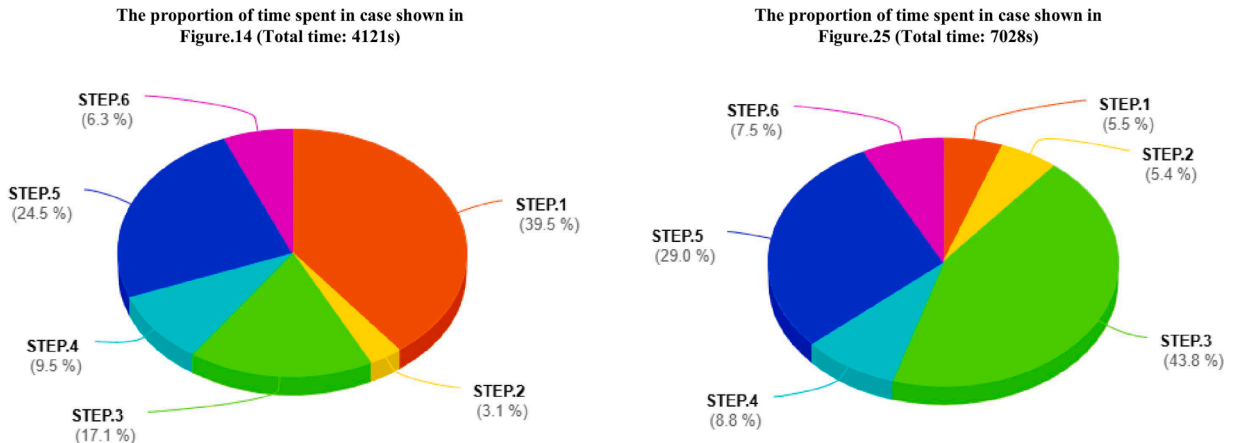


Fig. 33. The computing time compositions.

3. The setting of boundary conditions for process optimization is crucial for both structural evolution and slicing quality. However, since this setup is challenging to quantify and highly case-specific, we have not discussed it in detail. Based on our experience, we recommend that the positions of Γ_{start} and Γ_{end} should align as closely as possible with the Dirichlet boundary conditions used in structural optimization. The quality of curved layers improves, and the overall structural performance enhances as well.

For the future:

1. The printing example provided in the manuscript is relatively simple that may not fully demonstrate the advantages of multi-axis DED additive manufacturing. However, it is non-trivial to print with more complex topologies, since the thermal dissipation conditions for every spots are very different, which leads to potential local overheating and excessively large melt pool sizes, causing unpredictable issues such as structure minor collapse and insufficient height. Addressing this challenge by incorporating DED process simulation in design would be a necessary future work. In addition to shape precision, we also aim at achieving robust material property control through process manipulation.
2. Clearly, for traditional mechanical parts, the cost associated with multi-axis additive manufacturing is over high relative to the performance gains. Therefore, such manufacturing methods are more suitable for scenarios where performance improvement is prioritized over cost considerations. Aerospace components including internal cooling channels seem to be a promising application for our proposed approach. Hence, thermal-fluid-related problems will be targeted in our forthcoming work.
3. Curved slicing has significant effects on the physical properties or anisotropy of the printed materials. It is also necessary to consider this aspect in the optimization process, which is part of our future work.
4. For slicing path optimization, current algorithms still heavily rely on the boundary condition settings. In the future, we aim to incorporate boundary condition co-optimization to achieve better slicing results. Alternatively, leveraging our framework, we plan to integrate more advanced slicing optimization algorithms (like the works reported in [71,72]) to enhance the concurrent optimization performance.

CRediT authorship contribution statement

Shuzhi Xu: Writing – original draft, Visualization, Validation, Software, Methodology, Investigation, Conceptualization. **Jikai Liu:** Writing – review & editing, Validation, Resources, Project administration, Methodology, Funding acquisition, Formal analysis, Conceptualization. **Dong He:** Writing – original draft, Visualization, Validation, Software. **Kai Tang:** Writing – review & editing, Validation, Software, Resources. **Kentaro Yaji:** Writing – review & editing, Validation, Supervision, Resources, Investigation.

Declaration of competing interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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Appendix A. Sensitivity analysis

Recalling the optimization model formulated in Eq. (23):

$$\left\{ \begin{array}{l}
 \text{find : } \mu, \tau \\
 \text{minimize : } C_r(\mu) + L_0(\tau) \\
 \text{subject to : } \left\{ \begin{array}{l}
 \mathbf{K}(\mu)\mathbf{U} = \mathbf{F} \text{ (a1 - 1)} \\
 \mathbf{K}_{th}(\tau)\mathbf{T} = 0 \text{ (a1 - 2)} \\
 \mathbf{K}_{po}\boldsymbol{\varphi} = \mathbf{Q}_{po}(\mathbf{T}) \text{ (a1 - 3)} \\
 \frac{\sum_{e=1}^{N_E} \rho_e}{N_E} \leq V_f \text{ (a1 - 4)} \\
 \frac{\sum_{e=1}^{N_E} \bar{\chi}_e}{N_E} \leq V_{tol} \text{ (a1 - 5)} \\
 \mu_{min} \leq \mu_e \leq 1, \forall e \text{ (a1 - 6)} \\
 \tau_{min} \leq \tau_e \leq 1, \forall e \text{ (a1 - 7)}
 \end{array} \right.
 \end{array} \right. \quad (a1)$$

There are two design variables, one is μ for structure field, and another is τ for process field. Since we use the MMA solver for this optimization problem, the first gradient of the objective or constraints with respect to the design variables is required. Special care should be taken to treat the objective function properly. Especially for the process design variable τ , which only affect the second objective function $L_0(\tau)$. The corresponding Lagrangian function could be written as:

$$L = L_0 - \lambda_t^T \mathbf{R}_t - \lambda_p^T \mathbf{R}_p \quad (a2)$$

where $\mathbf{R}_t$ and $\mathbf{R}_p$ are two residual values for the control equations related to the $L_0(\tau)$:

$$\begin{aligned}
 \mathbf{R}_t &= \mathbf{K}_{th}(\tau)\mathbf{T} \\
 \mathbf{R}_p &= \mathbf{K}_{po}\boldsymbol{\varphi} - \mathbf{Q}_{po}(\mathbf{T})
 \end{aligned} \quad (a3)$$

Then, using the chain rule, the gradient of the L respect to the τ is:

$$\frac{dL}{d\tau} = \frac{\partial L_0}{\partial \varphi} \frac{d\varphi}{d\tau} - \lambda_t^T \frac{\partial \mathbf{R}_t}{\partial \mathbf{T}} \frac{d\mathbf{T}}{d\tau} - \lambda_t^T \frac{\partial \mathbf{R}_t}{\partial \tau} - \lambda_p^T \frac{\partial \mathbf{R}_p}{\partial \mathbf{T}} \frac{d\mathbf{T}}{d\tau} - \lambda_p^T \frac{\partial \mathbf{R}_p}{\partial \varphi} \frac{d\varphi}{d\tau} \quad (a4)$$

In the above equation, these two terms $(\frac{d\mathbf{T}}{d\tau}$ and $\frac{d\varphi}{d\tau})$ are hard to solve, so we could collect them as:

$$\frac{dL}{d\tau} = \left(\frac{\partial L_0}{\partial \varphi} - \lambda_p^T \frac{\partial \mathbf{R}_p}{\partial \varphi} \right) \frac{d\varphi}{d\tau} - \left(\lambda_t^T \frac{\partial \mathbf{R}_t}{\partial \mathbf{T}} + \lambda_p^T \frac{\partial \mathbf{R}_p}{\partial \mathbf{T}} \right) \frac{d\mathbf{T}}{d\tau} - \lambda_t^T \frac{\partial \mathbf{R}_t}{\partial \tau} \quad (a5)$$

Then, form the following two adjoint equations:

$$\frac{\partial L_0}{\partial \varphi} - \lambda_p^T \frac{\partial \mathbf{R}_p}{\partial \varphi} = 0 \quad (a6)$$

and

$$\lambda_t^T \frac{\partial \mathbf{R}_t}{\partial \mathbf{T}} + \lambda_p^T \frac{\partial \mathbf{R}_p}{\partial \mathbf{T}} = 0 \quad (a7)$$

we could avoid directly solving $\frac{d\mathbf{T}}{d\tau}$ and $\frac{d\varphi}{d\tau}$. Finally, the $\frac{dL}{d\tau}$ could be expressed as:

$$\frac{dL}{d\tau} = -\lambda_t^T \frac{\partial \mathbf{R}_t}{\partial \tau} \quad (a8)$$

According to the definition of $\mathbf{R}_t$ and $\mathbf{R}_p$, the adjoint equation Eq. (a6) could be expressed as:

$$\mathbf{K}_{po}\lambda_p = \frac{\partial L_0}{\partial \varphi} \quad (a9)$$

and once obtaining the adjoint value λ_p , it is used to solve the adjoint equation Eq. (a7), which could be further expressed as:

$$\mathbf{K}_{th}\lambda_p = -\lambda_t^T \frac{\partial \mathbf{Q}_{po}}{\partial T} \quad (\text{a10})$$

The derivation of other terms, especially those related to the structure variable, will study and easy to perform, and thus omitted here.

Data availability

Data will be made available on request.

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